ACAP
Alabama Comprehensive Assessment Program

SUMMATIVE

Grade 6
Mathematics

Alabama Educator Instructional Supports

Alabama Course of Study Standards

DATA RECOGNITION
DRC
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October 28, 2019

Introduction

The *Alabama Course of Study Instructional Supports: Math* is a companion manual to the 2016 *Revised Alabama Course of Study: Math* for Grades K–12. Instructional supports are foundational tools teachers may use to help students become independent learners as they build toward mastery of the *Alabama Course of Study* content standards.

- The purpose of the instructional supports found in this manual is to help teachers engage their students in exploring, explaining, and expanding their understanding of the content standards.

The content standards contained within the course of study may be accessed on the Alabama State Department of Education (ALSDE) website at www.alsde.edu.

Educators are reminded that content standards indicate minimum content—what all students should know and be able to do by the end of each grade level or course. Local school systems may have additional instructional or achievement expectations and may provide instructional guidelines that address content sequence, review, and remediation.

Organization

The organizational components of this manual include standards, guiding questions, connections to instructional supports, key academic terms, and examples of activities. The definition of each component is provided below:

Content Standard:	The content standard is the statement that defines what all students should know and be able to do at the conclusion of a given grade level or course. Content Standards contain minimum required content and complete the phrase "Students will."
Guiding Questions:	Each guiding question is designed to create a framework for the given standard. Therefore, each question is written to help teachers convey important concepts within the standard. By utilizing guiding questions, teachers are engaging students in investigating, analyzing, and demonstrating knowledge of the underlying concepts reflected in the standard.

Connection to Instructional Supports: The purpose of each instructional support is to engage students in exploring, explaining, and expanding their understanding of the content standards provided in the 2016 *Revised Alabama Course of Study: Math*. An emphasis is placed on the integration of the eight Standards for Mathematical Practice.

Mathematical Practices

The Standards for Mathematical Practice describe the varieties of expertise that mathematics educators at all levels should seek to develop in their students. They rest on the National Council of Teachers of Mathematics (NCTM) process standards and the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up: Helping Children Learn Mathematics.*

The Standards for Mathematical Practice are the same for all grade levels and are listed below.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Key Academic Terms: The academic terms included in each instructional support. These academic terms are derived from the standards and are to be incorporated into instruction by the teacher and used by the students.

Instructional Activities: A representative set of sample activities and examples that can be used in the classroom. The set of activities and examples is not intended to include all the activities and examples defined by the standard. These will be available in Fall 2020.

Additional Resources: Additional resources include resources that are aligned to the standard and may provide additional instructional strategies to help students build toward mastery of the designated standard. These will be available in Fall 2020.

Ratios and Proportional Relationships
Understand ratio concepts and use ratio reasoning to solve problems.
6.RP.1 Understand the concept of a ratio, and use ratio language to describe a ratio relationship between two quantities. Examples: “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”

Guiding Questions with Connections to Mathematical Practices:

What is a ratio?

M.P.6. Attend to precision. Describe a ratio as a quantitative comparison between two or more sets. Ratios can represent part-to-part and part-to-whole relationships. A ratio can be interpreted as a composed unit relating a whole to a part-to-part relationship that is repeatable. For example, 2 parts red paint and 3 parts blue paint can be combined to create a composed unit of a particular shade of purple paint. For all quantities of paint in this particular shade of purple, the ratio of red paint to blue paint will be some multiple of the ratio 2 to 3. Composed units can also be partitioned into parts that maintain the original ratio. For example, a pitcher of juice is made by combining 3 parts concentrate with 5 parts water. A glass of the juice would have reduced quantities of concentrate and water in the same ratio as the entire pitcher of juice.

When is it appropriate to describe a situation using ratios and ratio language?

M.P.3. Construct viable arguments and critique the reasoning of others. Create examples and counterexamples of ratio situations using ratio language. For example, use “for every” to describe the ratio 4:5 as “for every 4 dollars, you can buy 5 oranges” or a ratio of 4 dollars to 5 oranges. Additionally, the phrase “for every 2 yellow beads, there are 3 blue beads” is an example of a part-to-part ratio relationship of 2:3. However, the situation “Maria is twice as old as her brother Marco. Marco is 5 years old. How old will Maria be in 2 years?” cannot be represented with a ratio, as Maria will not always be twice as old as Marco.

M.P.4. Model with mathematics. Recognize and create a variety of models that illustrate ratio relationships. For example, a picture of 4 green cubes and 3 red cubes in a bag has a part-to-part ratio of 4 to 3. The same bag of cubes would have part-to-whole ratios of 4 green cubes to 7 total cubes or 3 red cubes to 7 total cubes.

Key Academic Terms:

ratio, quantity, relationship, part-to-part, part-to-whole, composed unit

Ratios and Proportional Relationships
Understand ratio concepts and use ratio reasoning to solve problems.
6.RP.2 Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio *a*:*b* with $b \neq 0$, and use rate language in the context of a ratio relationship. Examples: "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $\frac{3}{4}$ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (Expectations for unit rates in this grade are limited to noncomplex fractions.)

Guiding Questions with Connections to Mathematical Practices:

What is a unit rate?

M.P.6. Attend to precision. Describe a unit rate as a ratio expressed in the form a:b or $\frac{a}{b}$ where the value of b is 1. For example, the speed of a car is 55 miles per hour, or $\frac{\text{55 miles}}{\text{1 hour}}$.

M.P.4. Model with mathematics. Recognize and create a variety of models that illustrate ratio and unit rate relationships. For example, given the situation "A restaurant charges $8.00 for 4 chicken strips. How much does the restaurant charge for one chicken strip?" draw a visual representation to show the cost for one chicken strip.

How can unit rate relationships be described?

M.P.6. Attend to precision. Describe the meaning of unit rate and use vocabulary correctly when describing unit rates. For example, a can of soup costs $1.50 for 12 ounces, so the unit rate, which in this case means the number of dollars for exactly one ounce, is found using the proportion $\frac{\$1.50}{\text{12 ounces}} = \frac{\$0.125}{\text{1 ounce}} = \$0.125$ per ounce. The rate could also be thought of in terms of the quantity per dollar. The proportion would be $\frac{\text{12 ounces}}{\$1.50} = \frac{\text{8 ounces}}{\$1} = 8$ ounces per dollar.

Key Academic Terms:

ratio, rate, unit rate, quantity, relationship, part-to-part, part-to-whole

Ratios and Proportional Relationships
Understand ratio concepts and use ratio reasoning to solve problems.
6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **6.RP.3a** Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

Guiding Questions with Connections to Mathematical Practices:

How can patterns in ratio models be used to solve various ratio problems?

M.P.4. Model with mathematics. Identify and use patterns in ratio tables to solve various ratio problems. For example, given the problem "Julio makes green paint by mixing 4 parts blue paint to every 5 parts yellow paint. How many parts yellow paint should Julio use if he wants the same color of green paint and uses 8 parts blue paint?" solve by making a ratio table. Notice the additive pattern of adding 4 when moving down the table from blue to blue and adding 5 when moving down the table from yellow to yellow and the multiplicative pattern of multiplying by 1.25 when comparing blue to yellow.

How can plotting values from a table onto a coordinate plane help solve ratio problems?

M.P.1. Make sense of problems and persevere in solving them. Connect ratio tables and the patterns within them to their corresponding coordinate pairs on the coordinate plane. For example, notice that the coordinate pairs created by ratio tables always create straight lines.

Key Academic Terms:

ratio, rate, equivalent ratio, table, tape diagram, double number line diagram, equation, coordinate plane

Ratios and Proportional Relationships
Understand ratio concepts and use ratio reasoning to solve problems.
6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **6.RP.3b** Solve unit rate problems including those involving unit pricing and constant speed. Example: If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Guiding Question with Connections to Mathematical Practices:

How can unit rates be used to solve problems?

M.P.4. Model with mathematics. Demonstrate how a unit rate can be found using a variety of strategies. For example, to answer the questions "If 3 pounds of grapes cost $4.20, how much does 1 pound of grapes cost? How much do 5 pounds of grapes cost?" use a double number line to find the unit rate of $1.40 and use multiplicative thinking to find that 5 pounds of grapes cost $7.00.

Key Academic Terms:

unit rate, unit pricing, constant speed, equivalent ratio table, double number line, equation

Ratios and Proportional Relationships
Understand ratio concepts and use ratio reasoning to solve problems.
6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **6.RP.3c** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percent.

Guiding Questions with Connections to Mathematical Practices:

What is a percent and how is percent related to ratios?

M.P.6. Attend to precision. Understand the concept of percent and connect ratios and percent. For example, define a percent as a ratio that is always expressed out of 100, such as $27\% = \frac{27}{100}$.

How can ratio patterns be used to find common percents?

M.P.2. Reason abstractly and quantitatively. Find patterns and use ratio reasoning to solve percent problems. For example, to find 40% of 60, find 10% of 60, which is 6. Then multiply 6 by 4 to find 24 (40% is 4 times 10%).

What strategies can be used to find the missing part or whole in a percent problem?

M.P.1. Make sense of problems and persevere in solving them. Use a variety of strategies and models to solve percent problems that involve finding the whole given the percent, finding the part given the whole, and finding the part given the percent. For example, answer the question "What percent of 32 is 8?" by recognizing that 8 is $\frac{1}{4}$ of 32 and that 25 is $\frac{1}{4}$ of 100 to determine that 8 is 25% of 32.

Key Academic Terms:

percentage, per 100, quantity, whole, part

Ratios and Proportional Relationships
Understand ratio concepts and use ratio reasoning to solve problems.
6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **6.RP.3d** Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Guiding Questions with Connections to Mathematical Practices:

How is ratio reasoning used to convert measurement units within or between customary and metric systems?

M.P.5. Use appropriate tools strategically. Use a variety of strategies to show patterns of conversion between and within measurement systems. For example, using a centimeter/inch ruler simulates a double number line and can be used to find the conversion between inches and centimeters.

How can the proper unit be determined when solving conversion problems?

M.P.6. Attend to precision. Determine the correct unit when solving conversion problems by using the context. For example, given that 1 quart is approximately 0.95 liter, when finding how many quarts are in 3 liters, correctly identify the ending unit as quarts.

Key Academic Terms:

convert, conversion, measurement, customary system, metric system, ratio, table, unit

The Number System
Apply and extend previous understandings of multiplication and division to divide by fractions.
6.NS.4 Interpret and compute quotients of fractions, and solve word problems involving division of fractions, e.g., by using visual fraction models and equations to represent the problem. Examples: Create a story context for $(\frac{2}{3}) \div (\frac{3}{4})$, and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(\frac{2}{3}) \div (\frac{3}{4}) = \frac{8}{9}$ because $\frac{3}{4}$ of $\frac{8}{9}$ is $\frac{2}{3}$. (In general, $(\frac{a}{b}) \div (\frac{c}{d}) = \frac{ad}{bc}$.) How much chocolate will each person get if 3 people share $\frac{1}{2}$ lb of chocolate equally? How many $\frac{3}{4}$-cup servings are in $\frac{2}{3}$ of a cup of yogurt? How wide is a rectangular strip of land with length $\frac{3}{4}$ mi and area $\frac{1}{2}$ square mi?

Guiding Questions with Connections to Mathematical Practices:

How can the relationship between multiplication and division be used to solve fraction division problems?

M.P.7. Look for and make use of structure. Explain how to use the concepts of multiplication and division to solve fraction division problems and interpret the quotient. For example, when solving $\frac{3}{4} \div \frac{2}{3}$, relate it to "How many $\frac{2}{3}$ are in $\frac{3}{4}$?" or $\frac{3}{4} = a \times \frac{2}{3}$.

How can visual fraction models and context be used to solve fraction division problems?

M.P.4. Model with mathematics. Represent the quotient of two fractions with a visual model and/or context. For example, model $\frac{1}{2} \div \frac{1}{8}$ with the context of "Marco has $\frac{1}{2}$ of a pound of crackers. He makes bags of crackers that each weigh $\frac{1}{8}$ of a pound. How many bags of crackers does Marco make?" Use a tape diagram divided into eighths, then shade $\frac{1}{2}$, and then find the number of eighths that are in $\frac{1}{2}$, which is 4.

How are remainders interpreted in fraction division?

M.P.6. Attend to precision. Explain how the unit changes in fraction division and how that impacts the meaning of a remainder. For example, when solving $3\frac{1}{2} \div \frac{3}{4}$, shade $3\frac{1}{2}$ circles, where 1 circle is 1 unit. Then, split each circle into fourths. Now, $\frac{3}{4}$ is the unit, so every 3 of the one-fourth size parts is 1 unit. There are 4 full units, with 2 remaining out of 3 parts, so the solution is $4\frac{2}{3}$.

Why does multiplying by the reciprocal work when dividing fractions?

M.P.2. Reason abstractly and quantitatively. Develop, explain, and use the multiplying by the reciprocal rule, which states that dividing by a number is equivalent to multiplying by the reciprocal of that number, by using a variety of contexts and strategies, such as finding a common denominator and dividing across and the knowledge that fractions are division. For example, the quotient of $\frac{3}{4}$ and $\frac{2}{5}$ can be found by finding a common denominator and dividing across: $\frac{15}{20} \div \frac{8}{20} = \frac{15 \div 8}{20 \div 20} = \frac{15 \div 8}{1} = \frac{15}{8}$. Connect the process to using reciprocals to find a denominator of one: $\frac{\frac{3}{4} \times \frac{5}{2}}{\frac{2}{5} \times \frac{5}{2}} = \frac{\frac{15}{8}}{1} = \frac{15}{8}$ to show why invert and multiply works.

Key Academic Terms:

reciprocal, fraction model, unit

The Number System
Compute fluently with multi-digit numbers and find common factors and multiples.
6.NS.5 Fluently divide multi-digit numbers using the standard algorithm.

Guiding Questions with Connections to Mathematical Practices:

How are division algorithms related to each other?

M.P.8. Look for and express regularity in repeated reasoning. Develop the standard algorithm by looking for general methods from a concrete model and connecting the algorithm to partial quotients or other strategies, and evaluate the reasonableness of the solution. For example, use the standard algorithm to solve 336 ÷ 4, and evaluate the accuracy of the solution by using estimation or by multiplying the quotient by the divisor.

How can a remainder be interpreted?

M.P.8. Look for and express regularity in repeated reasoning. Recognize that the remainder in a quotient can be expressed as the numerator of a fraction, with the divisor as the denominator. For example, dividing 334 by 5 is asking the question "How many groups of size 5 are in 334?" There are 66 groups of size 5 with 4 remaining. The remainder of 4 can be interpreted as $\frac{4}{5}$ of a group of size 5. Therefore, there are $66\frac{4}{5}$ groups of size 5 in 334. The 4 remaining could also be interpreted as 0.8 or 80% of a group.

Key Academic Terms:

standard algorithm, dividend, divisor, remainder, fraction, fluency, factors, multiples

The Number System
Compute fluently with multi-digit numbers and find common factors and multiples.
6.NS.6 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Guiding Questions with Connections to Mathematical Practices:

How are operations with decimals similar to operations without decimals?

M.P.7. Look for and make use of structure. Extend previous knowledge of strategies, including place value, models, and the standard algorithm, to perform operations with multi-digit decimals. For example, the product of 2.4 and 1.35 can be determined by representing the numbers as whole numbers by multiplying by factors of 10 so that $2.4 \times 10 \times 1.35 \times 100 = 24 \times 135$. Multiply to find that $24 \times 135 = 3{,}240$, and then determine the final product by dividing by 1,000 (based on the factors used to convert to whole numbers: $10 \times 100 = 1{,}000$) for a solution of 3.24.

How can fractions be used to show that the standard algorithms for multiplication and division work with multi-digit decimals?

M.P.2. Reason abstractly and quantitatively. Verify that when numbers with multi-digit decimals are rewritten as fractions, the same products and quotients are generated as when using the standard algorithms. For example, when multiplying 2.4×1.35, 2.4 can be represented as $\frac{24}{10}$ and 1.35 can be represented as $\frac{135}{100}$. Therefore, 2.4×1.35 can be rewritten as $\frac{24}{10} \times \frac{135}{100} = \frac{3240}{1000} = 3.240$.

Key Academic Terms:

standard algorithm, operations, decimals, fractions, fluency, factors, multiples

The Number System
Compute fluently with multi-digit numbers and find common factors and multiples.
6.NS.7 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. Example: Express 36 + 8 as 4(9 + 2).

Guiding Questions with Connections to Mathematical Practices:

How are the greatest common factor and the distributive property used to rewrite a sum as a product?

M.P.7. Look for and make use of structure. Recognize that if the greatest common factor of two addends is identified, then each addend can be rewritten as a product that includes the greatest common factor and that the distributive property can then be applied to express the original sum as a multiple of the sum of two whole numbers with no common factor. For example, the numerical expression $8 + 14$ can be rewritten as $2 \times 4 + 2 \times 7$, which is equivalent to $2(4 + 7)$ after applying the distributive property. This is useful to make mental calculations easier, by adding 4 and 7 together and then doubling the sum.

How can the greatest common factor of two whole numbers be determined without listing all the factors of both numbers?

M.P.1. Make sense of problems and persevere in solving them. Recognize that the greatest common factor of two whole numbers can be determined using a variety of methods such as prime factorization or a ladder diagram. For example, the greatest common factor of 12 and 16 can be determined by first writing the prime factorization of 12 and 16, and then identifying the prime factors that both numbers have in common (e.g., $12 = 2 \times 2 \times 3$ and $16 = 2 \times 2 \times 2 \times 2$; therefore, the greatest common factor of 12 and 16 is 2×2 or 4).

Key Academic Terms:

greatest common factor, prime factorization, least common multiple, distributive property

The Number System
Apply and extend previous understandings of numbers to the system of rational numbers.
6.NS.8 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts explaining the meaning of 0 in each situation.

Guiding Questions with Connections to Mathematical Practices:

How do positive and negative numbers relate to 0?

M.P.2. Reason abstractly and quantitatively. Recognize that positive numbers are greater than 0 and may be represented with a (+) symbol or no symbol while negative numbers are less than 0 and are represented with a (−) symbol. For example, 4 is a positive number because it is 4 greater than 0, and −5 is a negative number because it is 5 less than 0.

What are opposites and how do they relate to 0?

M.P.2. Reason abstractly and quantitatively. Recognize that two numbers are opposites if they are the same distance from 0 and on different sides of 0. For example, −9 and 9 are opposites because they are both 9 away from 0 on a number line.

How does context change the interpretation of zero in problems with positive and negative numbers?

M.P.6. Attend to precision. Explain the meaning of zero when given a context. For example, zero in the context of money means “none,” whereas zero given the context of sea level does not mean “none” but refers instead to a location.

Key Academic Terms:

positive number, negative number, opposites

The Number System
Apply and extend previous understandings of numbers to the system of rational numbers.
6.NS.9 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **6.NS.9a** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.

Guiding Questions with Connections to Mathematical Practices:

What are opposites and how are they represented on a number line?

M.P.4. Model with mathematics. Represent a number and its opposite by showing that they are the same distance from 0 on a number line. For example, –2 and 2 are opposite numbers because they are both 2 units away from the point 0 on a number line.

What is the significance of the opposite of a number's opposite?

M.P.2. Reason abstractly and quantitatively. Recognize that the opposite of a number's opposite is the same as the number itself. For example, the opposite of 3 is –3, and the opposite of –3 is 3. Therefore, the opposite of the opposite of 3 is 3.

Key Academic Terms:

opposites, number line, rational number

The Number System
Apply and extend previous understandings of numbers to the system of rational numbers.
6.NS.9 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **6.NS.9b** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

Guiding Questions with Connections to Mathematical Practices:

What is the relationship between the signs of the coordinates of an ordered pair and the quadrant location of the ordered pair on the coordinate plane?

M.P.2. Reason abstractly and quantitatively. Recognize that the coordinate plane is divided into four quadrants created by the intersection of a horizontal line, the x-axis, and a vertical line, the y-axis, and that the signs of an ordered pair determine the quadrant location, such that (+, +) represents quadrant I, (–, +) represents quadrant II, (–, –) represents quadrant III, and (+, –) represents quadrant IV. For example, the ordered pair (–2, 2) is in quadrant II.

How is the location of an ordered pair affected when the sign of either coordinate is changed?

M.P.2. Reason abstractly and quantitatively. Recognize that changing the sign of the x-coordinate in an ordered pair results in a location that is a reflection across the y-axis, while changing the sign of the y-coordinate results in a location that is a reflection across the x-axis. For example, the ordered pair (–3, 5) is a reflection across the y-axis of the ordered pair (3, 5).

How does the number 0 in an ordered pair affect the location of the ordered pair?

M.P.2. Reason abstractly and quantitatively. Recognize that when one of the coordinates in an ordered pair is 0, then the location of the ordered pair is on an axis. For example, the ordered pair (0, 4) represents a point that is 4 units above the origin on the y-axis.

Key Academic Terms:

quadrant, coordinate plane, ordered pair, sign, y-axis, x-axis, axis, origin, rational number

The Number System
Apply and extend previous understandings of numbers to the system of rational numbers.
6.NS.9 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **6.NS.9c** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Guiding Questions with Connections to Mathematical Practices:

What is the relationship between the sign of a number and its location on a vertical or horizontal number line?

M.P.2. Reason abstractly and quantitatively. Recognize that negative numbers are located below zero on a vertical number line or to the left of zero on a horizontal number line, while positive numbers are located above zero on a vertical number line or to the right of zero on a horizontal number line. For example, the number $-\frac{3}{4}$ is located $\frac{3}{4}$ of a unit below zero on a vertical number line or $\frac{3}{4}$ of a unit left of zero on a horizontal number line.

What is the relationship between the signs of coordinates in an ordered pair and the location of an ordered pair on the coordinate plane?

M.P.2. Reason abstractly and quantitatively. Recognize that the first coordinate in an ordered pair indicates how far to move to the left (negative) or right (positive) from the origin, while the second indicates how far to move down (negative) or up (positive) from the origin. For example, the ordered pair (−3, 5.5) indicates a location that is 3 units left of the origin and 5.5 units up.

Key Academic Terms:

coordinate plane, ordered pair, sign, axis, origin, integer, rational number, horizontal, vertical, negative, positive

The Number System
Apply and extend previous understandings of numbers to the system of rational numbers.
6.NS.10 Understand ordering and absolute value of rational numbers. **6.NS.10a** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. Example: Interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.

Guiding Question with Connections to Mathematical Practices:

How can the position of numbers on a number line be used to compare numbers?

M.P.2. Reason abstractly and quantitatively. Understand that the numbers become greater when moving from left to right on a horizontal number line. For example, $-3 > -5$ because -3 is to the right of -5 on a number line, and $-2 < 0$ because -2 is to the left of 0 on a number line. Similarly, numbers become greater when moving up on a vertical number line.

Key Academic Terms:

inequality, number line, greater than, less than

The Number System
Apply and extend previous understandings of numbers to the system of rational numbers.
6.NS.10 Understand ordering and absolute value of rational numbers. **6.NS.10b** Write, interpret, and explain statements of order for rational numbers in real-world contexts. Example: Write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.

Guiding Question with Connections to Mathematical Practices:

How can statements of order and inequalities be used to represent comparisons in real-world contexts?

M.P.4. Model with mathematics. Model the relationship between measurements, such as elevation or temperature, with ordered lists or inequalities. For example, the inequality $-3.5 > -5$ can be used to show that the temperature of −3.5°C on Monday was warmer (greater) than the temperature of −5°C on Tuesday because −3.5 is above −5 on a vertical number line.

Key Academic Terms:

inequality, number line, greater than, less than, absolute value

The Number System
Apply and extend previous understandings of numbers to the system of rational numbers.
6.NS.10 Understand ordering and absolute value of rational numbers. **6.NS.10c** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. Example: For an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.

Guiding Questions with Connections to Mathematical Practices:

How can a number line be used to understand absolute value?

M.P.2. Reason abstractly and quantitatively. Recognize that the absolute value of a number is its distance from zero on a number line. For example, the absolute value of –3 is 3 because –3 is 3 units away from zero.

How is the absolute value of a number related to the absolute value of its opposite?

M.P.2. Reason abstractly and quantitatively. Understand that opposites have the same absolute value. For example, the absolute value of –5 is 5 and the absolute value of 5 is 5 because both are 5 units away from zero on a number line.

How does context impact the interpretation of an absolute value problem?

M.P.4. Model with mathematics. Interpret and solve a real-world problem using absolute value. For example, when given the problem "A whale has a recorded dive of –9,874 feet in relation to sea level. An airplane flies at 8,910 feet in relation to sea level. Is the whale or the airplane farther from sea level?" identify that the whale is farther than the airplane from sea level, which is represented by 0, since the absolute value of –9,874 is greater than the absolute value of 8,910. Also, a debt of 500 dollars, represented by –$500, has a greater magnitude than a credit of 50 dollars, represented by $50.

Key Academic Terms:

number line, absolute value, opposite, magnitude

The Number System
Apply and extend previous understandings of numbers to the system of rational numbers.
6.NS.10 Understand ordering and absolute value of rational numbers. **6.NS.10d** Distinguish comparisons of absolute value from statements about order. Example: Recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.

Guiding Question with Connections to Mathematical Practices:

How is comparing the absolute values of two numbers different from comparing the order of the two numbers?

M.P.2. Reason abstractly and quantitatively. Compare the absolute values of two numbers based on each number's distance from 0, or magnitude, and compare the order of two numbers based on each number's position relative to each other, left or right, on a horizontal number line. For example, consider –10 and 5. Since all negative numbers are less than all positive numbers, –10 is less than 5 and located to the left of 5 on a horizontal number line. However, |–10| is greater than |5| because –10 is farther away from 0.

Key Academic Terms:

absolute value, magnitude, constant

The Number System
Apply and extend previous understandings of numbers to the system of rational numbers.
6.NS.11 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Guiding Questions with Connections to Mathematical Practices:

How can a coordinate plane be used to solve real-word and mathematical problems?

M.P.5. Use appropriate tools strategically. Use a coordinate plane to solve a variety of problems. For example, a map of a park represents a basketball court as a rectangle on a coordinate plane. Three vertices of the rectangle are the points (–5, 2), (–5, –3), and (2, –3). Plot the points and the sides of the rectangle on the coordinate plane to find the rectangle's remaining vertex, (2, 2).

How does the absolute value of a coordinate help determine the distance between points on the coordinate plane?

M.P.4. Model with mathematics. Extend previous understanding of absolute value and magnitude to determine the distances between ordered pairs that contain negative coordinates. For example, recognize that the point (1, –8) is a greater distance from the x-axis than the point (1, 6) because the absolute value of –8 is greater than the absolute value of 6. Furthermore, the total distance between (1, –8) and (1, 6) is 14 units because the absolute value of –8 plus the absolute value of 6 is equivalent to the sum of 8 and 6.

Key Academic Terms:

coordinates, coordinate plane, absolute value, magnitude, quadrant

Expressions and Equations
Apply and extend previous understandings of arithmetic to algebraic expressions.
6.EE.12 Write and evaluate numerical expressions involving whole-number exponents.

Guiding Question with Connections to Mathematical Practices:

How can equivalent expressions be used to help write, evaluate, and compare expressions with whole-number exponents?

M.P.7. Look for and make use of structure. Express repeated multiplication by writing expressions using exponents. For example, when finding the volume of a cube with side lengths of 6 units, write the expression $6 \times 6 \times 6$ as 6^3 to represent the volume in cubic units.

M.P.7. Look for and make use of structure. Represent expressions with whole-number exponents by applying the properties of operations and order of operations to create equivalent expressions. For example, evaluate $4^3 - 3(2 + 4^2) + 8$ by using the order of operations and the distributive property to find the solution 18.

Key Academic Terms:

exponent, base, order of operations

Expressions and Equations
Apply and extend previous understandings of arithmetic to algebraic expressions.
6.EE.13 Write, read, and evaluate expressions in which letters stand for numbers. **6.EE.13a** Write expressions that record operations with numbers and with letters standing for numbers. Example: Express the calculation, “Subtract y from 5,” as $5 – y$.

Guiding Questions with Connections to Mathematical Practices:

What does a variable represent in an expression?

M.P.6. Attend to precision. Recognize that in an expression, a variable represents a quantity that can change. For example, if there are 5 groups of students in a classroom, and each group has n students, the expression $5n$ represents the total number of students in the classroom.

How can situations be represented with expressions using variables?

M.P.4. Model with mathematics. Represent real-world and mathematical situations with an expression that contains variables. For example, given that 1 hat costs $26, 2 hats cost $52, 3 hats cost $78, and the same pattern continues, write the expression $26h$ to indicate the total cost (in dollars) of h hats.

Key Academic Terms:

expression, variable, operation

Expressions and Equations
Apply and extend previous understandings of arithmetic to algebraic expressions.
6.EE.13 Write, read, and evaluate expressions in which letters stand for numbers. **6.EE.13b** Identify parts of an expression using mathematical terms (*sum, term, product, factor, quotient, coefficient*); view one or more parts of an expression as a single entity. Example: Describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.

Guiding Questions with Connections to Mathematical Practices:

How are parts of expressions defined with mathematical terms?

M.P.7. Look for and make use of structure. Represent parts of an expression using mathematical terms for various operations. For example, in the expression $8(4 + p) \div 2$, 8 and $(4 + p)$ are factors of the product $8(4 + p)$, $(4 + p)$ is a single entity that is the sum of two terms, and the entire expression is a quotient with a dividend of $8(4 + p)$ and a divisor of 2.

What distinguishes a term in an expression from the expression itself?

M.P.6. Attend to precision. Recognize that terms are parts of an expression that are either added together or subtracted from each other. For example, the expression $3a + 2b$ contains two terms, $3a$ and $2b$, because they are added together.

What distinguishes a coefficient from a constant in an algebraic expression?

M.P.6. Attend to precision. Recognize that a coefficient is a number multiplied by a variable, while a constant is a quantity that does not change. For example, in the expression $4c + 1$, 4 is a coefficient and 1 is a constant.

How can a context help create meaning for an algebraic expression?

M.P.2. Reason abstractly and quantitatively. Interpret an expression and a context to give meaning to the different parts of the expression. For example, given that Rebecca picks a total of $3h$ pints of blueberries for a period of h hours, interpret the 3 as the number of pints of blueberries Rebecca picks each hour, h.

Key Academic Terms:

expression, algebraic expression, variable, operation, term, coefficient, constant, sum, product, quotient, factor

Expressions and Equations
Apply and extend previous understandings of arithmetic to algebraic expressions.
6.EE.13 Write, read, and evaluate expressions in which letters stand for numbers. **6.EE.13c** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). Example: Use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.

Guiding Questions with Connections to Mathematical Practices:

How are variables related to mathematical formulas?

M.P.4. Model with mathematics. Use a formula to represent the relationship between quantities that can change (variables) in an expression or equation and know that variables represent specific attributes. For example, the formula $A = l \times w$ shows the relationship between the area, A, of a rectangle and the product of the rectangle's length, l, and width, w.

How can an expression be evaluated for specific values of their variables?

M.P.1. Make sense of problems and persevere in solving them. Use properties of operations and order of operations to evaluate an expression for specific values of the variables. For example, to evaluate $3(2 + z) - 6 \div 2$ where $z = 4$, decide whether to distribute the 3 in $3(2 + z)$ or to substitute the 4 for z and add within the grouping symbols first, using the order of operations. Then continue solving using properties of operations and order of operations for a solution of 15.

Why is the conventional order of operations significant when using formulas?

M.P.4. Model with mathematics. Recognize that formulas are constructed by assuming the conventional order of operations. For example, the formula for the surface area of a cube with side length s, $A = 6s^2$, only works when the exponent is evaluated prior to multiplying by 6. If $s = 2$ inches, then the surface area equals 24 square inches rather than 144 square inches.

Key Academic Terms:

variable, formula, order of operations, expression, parentheses, exponent

Expressions and Equations
Apply and extend previous understandings of arithmetic to algebraic expressions.
6.EE.14 Apply the properties of operations to generate equivalent expressions. Example: Apply the distributive property to the expression 3(2 + *x*) to produce the equivalent expression 6 + 3*x*; apply the distributive property to the expression 24*x* + 18*y* to produce the equivalent expression 6(4*x* + 3*y*); apply properties of operations to *y* + *y* + *y* to produce the equivalent expression 3*y*.

Guiding Questions with Connections to Mathematical Practices:

How can different symbols representing multiplication and division be used to generate equivalent expressions?

M.P.6. Attend to precision. Recognize and use parentheses, a juxtaposition, or a dot to represent multiplication and a division or fraction bar to represent division. For example, recognize that 3 times c can be expressed as $3(c) = 3c = 3 \bullet c$ and that 4 divided by y can be expressed as $4 \div y = \frac{4}{y} = 4 \bullet \frac{1}{y} = 4\left(\frac{1}{y}\right)$.

How are expressions with variables composed and decomposed?

M.P.7. Look for and make use of structure. Use the properties of operations to compose and decompose expressions with variables. For example, decompose $12x$ in a variety of ways, such as $x + x + x + x + x + x + x + x + x + x + x + x$, $2 \bullet 6x$, $3(4x)$, $6x + 6x$, and $x(10 + 2)$.

M.P.6. Attend to precision. Identify conventions when using variables, such as hidden ones, combining like terms, and the various multiplication representations. For example, identify the expression $7p - p$ as $7p - 1p$ to create the equivalent expression $6p$.

Key Academic Terms:

expression, equivalent, variable, like terms, properties of operations

Expressions and Equations
Apply and extend previous understandings of arithmetic to algebraic expressions.
6.EE.15 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). Example: The expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y represents.

Guiding Question with Connections to Mathematical Practices:

How can the equivalency of two algebraic expressions be verified without substituting a value into the variables?

M.P.2. Reason abstractly and quantitatively. Apply any of the properties of operations to one expression so that it generates a second expression. For example, the expression $2(5k + 1)$ is equivalent to the expression $10k + 2$ because $2(5k + 1)$ becomes $10k + 2$ when the distributive property is applied.

Key Academic Terms:

equivalent expressions, properties of operations, variable

Expressions and Equations
Reason about and solve one-variable equations and inequalities.
6.EE.16 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Guiding Question with Connections to Mathematical Practices:

What does it mean to solve an equation or inequality?

M.P.6. Attend to precision. Define solving an equation or inequality as the process of reasoning, using substitution or the structure of a problem, to find the numbers which make that equation or inequality true. For example, 6 is the solution to the equation $w + 4 = 10$ because the equation is true when w is 6; and in the inequality $3z < 24$, the solution is $z < 8$ because the inequality is true with any number less than 8.

Key Academic Terms:

equation, inequality, solution, substitution

Expressions and Equations
Reason about and solve one-variable equations and inequalities.
6.EE.17 Use variables to represent numbers, and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number or, depending on the purpose at hand, any number in a specified set.

Guiding Questions with Connections to Mathematical Practices:

What is a variable and how can it be used to solve problems?

M.P.2. Reason abstractly and quantitatively. Recognize that a variable is a letter or symbol representing a number and that an expression containing a variable or variables can be used to solve problems. For example, if Elizabeth practices the violin for 25 minutes every day, then the variable d could be used to represent the number of days Elizabeth practiced the violin and the expression $25d$ would represent the number of minutes Elizabeth practiced over any number of days. To determine how many minutes Elizabeth would practice the violin in a week, the equation $25d$ can be changed to $25(7) = 175$ minutes for the week.

M.P.7. Look for and make use of structure. Recognize that when the same variable is used more than once within an expression, the value of the variable is the same in each instance. For example, each x in the expression $2x + 4 = 3(x - 2)$ has the same value.

How can the meaning of the variable be determined for a given problem?

M.P.6. Attend to precision. Understand that a variable can be a single value, or all the values in a solution set, and that the solution set may be limited by a context. For example, when given the expression $0.89b$ to calculate the cost of bananas for b pounds of bananas, know the solution set is all values greater than or equal to 0, as it is not reasonable to have negative pounds of bananas.

Key Academic Terms:

variable, expression, equation, solution set

Expressions and Equations
Reason about and solve one-variable equations and inequalities.
6.EE.18 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.

Guiding Questions with Connections to Mathematical Practices:

How are inverse operations one of many useful strategies in solving equations?

M.P.1. Make sense of problems and persevere in solving them. Recognize that addition and subtraction "undo" one another when solving for a variable because they are inverse operations, and recognize that multiplication and division are inverse operations as well. For example, in the equation $x + 2 = 3$, the variable x can be isolated by subtracting 2 from both sides of the equation because subtracting 2 is the inverse of adding 2, and the equation is still balanced. Similarly, in the equation $3x = 6$, the variable x can be isolated by dividing both sides of the equation by 3 because dividing by 3 is the inverse of multiplying by 3.

How can a variable be used to represent an unknown quantity in real-world and mathematical problems?

M.P.2. Reason abstractly and quantitatively. Represent a situation symbolically, attending to the meaning of the variable. Solve the written equation, and make sense of the solution in the context of the situation. For example, represent the situation "Maeva had $36.48. Her friend gave her a jar full of coins, and then Maeva had $52.98," with the equation $36.48 + x = 52.98$, where x is the amount of money in the coin jar. Solving the equation gives $x = 16.5$, which means that Maeva's friend gave her $16.50 in coins. The answer makes sense when estimation, subtraction, or the context of the problem is used to check it.

Key Academic Terms:

equation, variable, inverse operation, unknown, isolate

Expressions and Equations
Reason about and solve one-variable equations and inequalities.
6.EE.19 Write an inequality of the form *x* > *c* or *x* < *c* to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form *x* > *c* or *x* < *c* have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Guiding Questions with Connections to Mathematical Practices:

How does an inequality compare to an equation?

M.P.2. Reason abstractly and quantitatively. Recognize that equations and inequalities are both mathematical sentences, where “=” indicates that two quantities are equal, “<” indicates the first number is less than the second, and “>” indicates that the first number is greater than the second. For example, the mathematical sentence $x = \frac{6}{5}$ indicates that x has the same value as $\frac{6}{5}$, while the mathematical sentence $\frac{8}{4} < p$ indicates that the value of p is any and all values greater than $\frac{8}{4}$.

What is the meaning of a solution set in an inequality?

M.P.2. Reason abstractly and quantitatively. Understand that an inequality may have more than one solution. An inequality can have many solutions, and those solutions are called a solution set. For example, the inequality $x > 3$ has infinitely many solutions in its solution set because there are an infinite number of values that are larger than 3.

M.P.6. Attend to precision. Represent a real-world solution set of an expression with an inequality. For example, when given the context “Sara runs 7.4 feet every s seconds,” the solution set would be $s > 0$ because Sara can only run for more than 0 seconds, and there would be a reasonable upper limit to the solution as well because Sara cannot run for infinite positive seconds.

How are number lines useful for representing the solution set of an inequality?

M.P.4. Model with mathematics. Represent more than one solution, including infinitely many solutions, with an open circle and shading or arrow on a number line. For example, the solution set of the inequality $x < 4$ can be shown on a number line with an open circle at 4 and shading and an arrow to the left of 4.

How do you know if a solution is feasible?

M.P.3. Construct viable arguments and critique the reasoning of others. Understand that solutions to inequalities modeling real-world situations may include answers that are not accurate in the context of the situation. For example, 23 members of a local choir will be going on a trip. The choir will be taking vans that will each carry up to 5 choir members from the group. Write an inequality to represent the number of vans, v, the choir can take. The inequality $v > 23 \div 5$ can be written, or $v > 4.6$. While $v = 5.5$ is a solution to the inequality $v > 4.6$, 5.5 vans is not a feasible solution because it is not possible to take 0.5 van.

Key Academic Terms:

inequality, equation, solution set, equal, greater than, less than, open circle, number line, infinitely many, feasible

Expressions and Equations
Represent and analyze quantitative relationships between dependent and independent variables.
6.EE.20 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. Example: In a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

Guiding Questions with Connections to Mathematical Practices:

What do independent and dependent variables represent in an equation?

M.P.2. Reason abstractly and quantitatively. Identify the independent variable as the “input” of an equation and the dependent variable as the “output” of an equation. Show that the value of the dependent variable is contingent on (depends on) the value of the independent variable. For example, if the equation $p = 0.75m$ is used to represent a total number of pages read, p, after m minutes, then m is the independent variable and p is the dependent variable. The number of pages read depends on the number of minutes spent reading.

How can the meaning of the variables be used to determine the variable that is independent and the variable that is dependent?

M.P.1. Make sense of problems and persevere in solving them. Analyze the problem to define the meaning of each variable to help discern which variable is independent and which is dependent. For example, given the situation “Imani buys one box of pencils that costs $2.99 and n notebooks that cost $1.89 per notebook. She spends a total of t dollars for the box of pencils and notebooks,” determine that n is the number of notebooks and is the independent variable, and t is the total cost, dependent on the number of notebooks, so it is the dependent variable.

How can the relationship between an independent variable and a dependent variable be expressed in a graph or table?

M.P.4 Model with mathematics. Recognize that a table or graph can be used to show the same relationship between an independent variable and a dependent variable that is expressed in an equation. For example, a table indicating that 3 ice cream cones cost $4.50 and 5 ice cream cones cost $7.50 represents the same relationship as the equation $p = 1.5c$, where the dependent variable, p, is the total cost of c ice cream cones.

Key Academic Terms:

independent variable, dependent variable, variable, equation, table, graph

Geometry
Solve real-world and mathematical problems involving area, surface area, and volume.
6.G.21 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Guiding Questions with Connections to Mathematical Practices:

How can the formula for the area of a rectangle be used to find the area of a parallelogram?

M.P.1. Make sense of problems and persevere in solving them. Use a concrete model to visualize a triangular portion from one side of a parallelogram "cut off" and repositioned on the other side of the parallelogram to create a rectangle so that the base of the parallelogram equals the length of the new rectangle and the height of the parallelogram equals the width of the new rectangle. For example, a parallelogram with a base of 10 inches and a height of 5 inches can be changed to create a rectangle with a length of 10 inches and a width of 5 inches that has an equivalent area.

How can the formula for the area of a parallelogram be used to find the area of a triangle?

M.P.1. Make sense of problems and persevere in solving them. Use a concrete model to recognize that a parallelogram with base b and height h can be divided into two equal triangles each with base b and height h, so the area of a triangle with base b and height h is half the area of a parallelogram with base b and height h. For example, the area of a parallelogram with a base of 8 centimeters and a height of 4 centimeters is 8×4, so the area of a triangle with a base of 8 centimeters and a height of 4 centimeters is half of 8×4.

How can the area of a composite figure be determined using the area of smaller figures?

M.P.1. Make sense of problems and persevere in solving them. Recognize that if a composite figure is decomposed into smaller figures, then the area of the composite figure is equal to the sum of the areas of the smaller figures. For example, the area of an L-shaped patio is equal to the sum of the areas of the two rectangles that combine to make the L-shaped patio.

Key Academic Terms:

area, triangles, quadrilaterals, polygons, decomposition, composite, base, height

Geometry
Solve real-world and mathematical problems involving area, surface area, and volume.
6.G.22 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

Guiding Question with Connections to Mathematical Practices:

How is the process of determining the volume of prisms with fractional edge lengths similar to the process of determining the volume of prisms with whole-number edge lengths?

M.P.1. Make sense of problems and persevere in solving them. Extend previous understandings of how to calculate the volume of prisms with whole number edge lengths to prisms with fractional edge lengths, and use manipulatives to better understand volume. For example, the process for determining the volume of a prism with a length of $\frac{1}{2}$ centimeter, a width of $\frac{1}{3}$ centimeter, and a height of $\frac{1}{4}$ centimeter is the same as determining the volume of a prism with a length of 2 centimeters, a width of 3 centimeters, and a height of 4 centimeters.

Key Academic Terms:

volume, prism, edge, right rectangular prism

Geometry
Solve real-world and mathematical problems involving area, surface area, and volume.
6.G.23 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Guiding Questions with Connections to Mathematical Practices:

How can the coordinate plane be used to determine the side length of figures?

M.P.6. Attend to precision. Utilize the axes on the coordinate plane as a measurement tool to determine side length. For example, if the vertices of the base of a triangle are plotted at (–3, 0) and (4, 0), then the length of the base can be determined as 7 units by counting the number of tick marks from –3 to 4 along the *x*-axis.

How can the attributes of shapes be used to classify a figure on a coordinate plane?

M.P.3. Construct viable arguments and critique the reasoning of others. Use the coordinate plane and the characteristics of shapes to justify the shape of a drawn figure. For example, a quadrilateral with one set of opposite sides having lengths of 5 units, the other set of opposite sides having lengths of 3 units, and all angles being right angles must be a rectangle.

Key Academic Terms:

vertices, coordinates, coordinate plane, axis, attributes

Geometry
Solve real-world and mathematical problems involving area, surface area, and volume.
6.G.24 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Guiding Question with Connections to Mathematical Practices:

What is a net and how is it useful for determining the surface area of a three-dimensional figure?

M.P.4. Model with mathematics. Recognize that a three-dimensional figure can be “flattened” or “unfolded” to create a two-dimensional figure called a net that shows each face simultaneously, and that the sum of the areas of each face is equal to the entire surface area. For example, the net of a cube with side length 3 units consists of 6 faces that each have an area of 9 units, making the surface area of the cube 54 square units.

Key Academic Terms:

surface area, net, three-dimensional

Statistics and Probability
Develop understanding of statistical variability.
6.SP.25 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. Example: “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.

Guiding Questions with Connections to Mathematical Practices:

How is variability affected by the way a question is posed?

M.P.2. Reason abstractly and quantitatively. Understand that the wording or framing of a question affects whether there is variation in the answers. For example, asking people how many state capitals there are in the US does not allow for variation, but asking people how many state capitals they have visited does allow for variation in the data set.

How is variability shown in a data set?

M.P.6. Attend to precision. Understand that variability refers to a spread of values in a data set. For example, the variability of the data set {1, 2, 3, 4, 5} is greater than the variability of the data set {1, 1, 1, 1, 2}.

Key Academic Terms:

variation, data, spread, statistical question, variability

Statistics and Probability
Develop understanding of statistical variability.
6.SP.26 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

Guiding Question with Connections to Mathematical Practices:

How can distribution be used to describe a data set?

M.P.7. Look for and make use of structure. Interpret the distribution of data in terms of its center, spread, and overall shape. For example, describe a dot plot of exam scores as being skewed left, skewed right, or symmetrical.

Key Academic Terms:

distribution, frequency, center, spread, shape, skew, symmetrical, cluster, peak, gap

Statistics and Probability
Develop understanding of statistical variability.
6.SP.27 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Guiding Questions with Connections to Mathematical Practices:

What does the mean of a data set represent?

M.P.2. Reason abstractly and quantitatively. Understand that the mean is a measure of center that represents the equal distribution of the data, in the sense of unit rate, that can be calculated by finding the sum of all items in a data set divided by the number of items in the data set. For example, the number of books a group of students read during spring break can be represented by the data set {2, 3, 4, 5, 5, 6, 7, 8}. The mean of this data set equals $\frac{40}{8}$, or 5, and represents how many books each student would have read if they had all read the same number of books.

What does the median of a data set represent?

M.P.2. Reason abstractly and quantitatively. Understand that the median is a measure of center that represents the middle value of a data set that is in numerical order (or the mean of the two middle values with an even number of items). For example, a group of seven students were asked to roll a number cube until they get a 1 and to count the number of rolls. The data set {8, 9, 6, 1, 17, 2, 5} represents the number of rolls it took each of the students to get a 1. The median of this data set can be determined by rearranging the data into numerical order, which is {1, 2, 5, 6, 8, 9, 17}, and identifying that 6 is the middle value. The same number of students took more than 6 rolls as the number of students who took less than 6 rolls.

What does the range of a data set represent and how is it different from measures of center?

M.P.2. Reason abstractly and quantitatively. Understand that unlike measures of center, the range indicates variability within a data set by accounting for the difference between the greatest value and the smallest value. For example, consider the following data sets that represent the number of baskets scored by Molly and Kim over a five-game period. The number of baskets Molly scored were {10, 12, 12, 12, 14}, and the number of baskets Kim scored were {4, 8, 12, 16, 20}. Both data sets have a mean of 12; however, the number of baskets Kim scores in a game has more variability than the number of baskets Molly scores in the game because the range for Kim is 4 times as large as the range for Molly.

What does the interquartile range of a data set represent?

M.P.2. Reason abstractly and quantitatively. Understand that interquartile range is the distance between the first and third quartiles of a data set and is a measure of variability. Consider again the data set generated by the students rolling a number cube until they got a 1, {1, 2, 5, 6, 8, 9, 17}. The first quartile is 2 and the third quartile is 9, so the interquartile range of this data set would be $9 - 2 = 7$. This means that the middle 50% of the data can be found between 2 and 9 with a range of 7.

Key Academic Terms:

mean, median, mode, range, quartile, interquartile range (IQR), measure of center, measure of variation, outlier, data set

Statistics and Probability
Summarize and describe distributions.
6.SP.28 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

Guiding Questions with Connections to Mathematical Practices:

What does the term "frequency" indicate and how can it be represented?

M.P.5. Use appropriate tools strategically. Understand that the frequency of a specific data value refers to the number of times it occurs within a data set and that such information can be organized and displayed on a dot plot or histogram. For example, a gardener counted the number of flower buds on 7 rosebushes he was growing and recorded the information in the following list: {1, 2, 3, 3, 1, 2, 2}. The frequency for each number of flower buds can be represented with a dot plot in which dots are placed over points on a number line to represent that there are two rosebushes with 1 flower bud, three rosebushes with 2 flower buds, and two rosebushes with 3 flower buds.

What is a histogram and how does it show frequency differently than a dot plot?

M.P.5. Use appropriate tools strategically. Understand that while a dot plot displays the frequency of each individual value in a data set, a histogram uses a bar to display values within equal intervals. For example, when displaying the ages of attendees at a fair, a histogram might display the data values within the intervals 0 to 9, 10 to 19, 20 to 29, 30 to 39, and so on, instead of displaying data for each individual age within those intervals.

What is a box plot and how does it represent data?

M.P.5. Use appropriate tools strategically. Interpret the range, median, interquartile range, first and third quartiles, and outliers as a box plot for a data set. For example, the number of times students rolled a number cube until they got a 1 is {1, 2, 5, 6, 8, 9, 17}. The median of this data set is 6, the first quartile is 2, and the third quartile is 9. To construct a box plot for this data set, draw a number line from 1 to 17, mark the median and quartiles above the number line, and make a box to show the interquartile range from 2 to 9. Finish the box plot by drawing the range as lines that go down to the minimum, 1, and up to the maximum, 17.

Key Academic Terms:

variation, data, spread, statistical question, frequency, dot plot, histogram, box plot, outlier, range, minimum, maximum

Statistics and Probability
Summarize and describe distributions.
6.SP.29 Summarize numerical data sets in relation to their context, such as by: **6.SP.29a** Reporting the number of observations.

Guiding Questions with Connections to Mathematical Practices:

How can the total number of observations in a data set be determined?

M.P.6. Attend to precision. Identify how data is organized and accounted for in dot plots, frequency tables, and histograms. For example, if a data set includes 10 total observations, then a dot plot or frequency table representing the data will include exactly 10 dots or 10 tally marks.

Why is the number of observations important when interpreting a data set in context?

M.P.1. Make sense of problems and persevere in solving them. Explain that the number of observations provides insight into the appropriateness of the size of the data set for a context and is needed to make comparisons within the data set. For example, a data set that is meant to represent the heights of all 6th graders in a large middle school would not be appropriate if it only included the heights of 5 students.

Key Academic Terms:

observations, data set, dot plots, frequency tables, histograms

Statistics and Probability
Summarize and describe distributions.
6.SP.29 Summarize numerical data sets in relation to their context, such as by: **6.SP.29b** Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

Guiding Question with Connections to Mathematical Practices:

How do the context and units of measurement of collected data affect the summary and display of numerical data?

M.P.6. Attend to precision. Recognize that an accurate summary of data requires consistency and precision when collecting the data. For example, if a researcher wants to display the water depths of lakes in a county, then he or she must measure the depths with a consistent unit and under similar conditions (e.g., the same time of year) every time.

Key Academic Terms:

data, attribute, unit of measurement

Statistics and Probability
Summarize and describe distributions.
6.SP.29 Summarize numerical data sets in relation to their context, such as by: **6.SP.29c** Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation) as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

Guiding Questions with Connections to Mathematical Practices:

What does an outlier represent and how can it affect measures of center or variability?

M.P.4. Model with mathematics. Recognize that an outlier refers to a value that is inconsistent with the data set because it is either much greater or much less than the other values in the data set and that such values can cause some measures of center or variability to be unrepresentative. Use context to determine the importance of the outlier. For example, if the prices of 6 bikes on a rack are {$85, $90, $105, $110, $120, $360}, then the value $360 is an outlier because it is a much higher price than all other values and skews the mean so much that it no longer represents the other values.

What are quartiles and how do they divide data?

M.P.4. Model with mathematics. Understand that quartiles represent three values, including the median, that are used to divide a set of data into fourths. For example, if seven people are asked how long they have had their current car, the data set may consist of the values {2, 4, 6, 8, 10, 10, 14}. The three quartiles are 4 years (first quartile), 8 years (median), and 10 years (third quartile).

What is the interquartile range and how does it differ from the range?

M.P.2. Reason abstractly and quantitatively. Recognize that while the range indicates variability within a data set by accounting for the difference between the greatest value and the smallest value, the interquartile range indicates variability within a data set by accounting for the difference between the third (upper) quartile and the first (lower) quartile. Often, the interquartile range helps to summarize the measure of spread better than the range since it is less affected by outliers than the range. For example, if seven people are asked how long they have had their current car, the data set may consist of the values {2, 4, 6, 8, 10, 10, 14}; the range equals 12, while the interquartile range equals 6.

What is the mean absolute deviation of a data set and how does it differ from the mean?

M.P.4. Model with mathematics. Recognize that while the mean is a measure of center, the mean absolute deviation is a measure of variation that indicates the mean amount that the data differ from the mean of the data. For example, the data set {4, 4, 6, 8, 8, 12, 14} represents the ages of children at a restaurant on one occasion. The mean age of the children is 8, while the mean absolute deviation of the ages is approximately 2.86, which indicates that, on average, each child's age is within 2.86 years of 8.

Key Academic Terms:

measure of center, measure of variability, mean, median, interquartile range (IQR), outlier, mean absolute deviation

Statistics and Probability
Summarize and describe distributions.
6.SP.29 Summarize numerical data sets in relation to their context, such as by: **6.SP.29d** Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Guiding Questions with Connections to Mathematical Practices:

When does the mean provide the best summary of data?

M.P.3. Construct viable arguments and critique the reasoning of others. Recognize that the mean is appropriate for data that are normally distributed. For example, the mean is an appropriate measure of center for the data set representing the number of hits a baseball player had over an eight-game stretch, {2, 3, 1, 4, 3, 1, 3, 2}, because there are no outliers or clusters.

When does the median provide the best summary of data?

M.P.3. Construct viable arguments and critique the reasoning of others. Recognize that the median is appropriate for data that are not normally distributed. For example, the median is an appropriate measure of center for the data set representing the number of letters or characters found in different languages, {26, 28, 24, 33, 29, 21, 3,000}, because the mean is significantly affected by the outlier of 3,000.

How do context and the shape of a data distribution help determine the measure of variability to use to describe a data set?

M.P.3. Construct viable arguments and critique the reasoning of others. Use context and the shape of a data distribution to determine the measure of variability to describe a data set. For example, if the data set has extreme outliers, it makes sense to use the median for the measure of center because it is less sensitive to extreme values, and, therefore, it also makes sense to use the interquartile range for the measure of variability.

Key Academic Terms:

measure of center, measure of variability, mean, median, mode, outlier