

# Grade 8 Mathematics

# Alabama Educator Instructional Supports

**Alabama Course of Study Standards** 





#### Introduction

The *Alabama Course of Study Instructional Supports: Math* is a companion manual to the 2016 *Revised Alabama Course of Study: Math* for Grades K–12. Instructional supports are foundational tools teachers may use to help students become independent learners as they build toward mastery of the *Alabama Course of Study* content standards.

 The purpose of the instructional supports found in this manual is to help teachers engage their students in exploring, explaining, and expanding their understanding of the content standards.

The content standards contained within the course of study may be accessed on the Alabama State Department of Education (ALSDE) website at <a href="https://www.alsde.edu">www.alsde.edu</a>.

Educators are reminded that content standards indicate minimum content—what all students should know and be able to do by the end of each grade level or course. Local school systems may have additional instructional or achievement expectations and may provide instructional guidelines that address content sequence, review, and remediation.

### **Organization**

The organizational components of this manual include standards, guiding questions, connections to instructional supports, key academic terms, and examples of activities. The definition of each component is provided below:

Content Standard:

The content standard is the statement that defines what all students should know and be able to do at the conclusion of a given grade level or course. Content Standards contain minimum required content and complete the phrase "Students will."

Guiding

Questions:

Each guiding question is designed to create a framework for the given standard. Therefore, each question is written to help teachers convey important concepts within the standard. By utilizing guiding questions, teachers are engaging students in investigating, analyzing, and demonstrating knowledge of the underlying concepts reflected in the standard.

# Connection to Instructional Supports:

The purpose of each instructional support is to engage students in exploring, explaining, and expanding their understanding of the content standards provided in the 2016 *Revised Alabama Course of Study:*Math. An emphasis is placed on the integration of the eight Standards for Mathematical Practice.

#### Mathematical Practices

The Standards for Mathematical Practice describe the varieties of expertise that mathematics educators at all levels should seek to develop in their students. They rest on the National Council of Teachers of Mathematics (NCTM) process standards and the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up: Helping Children Learn Mathematics*.

The Standards for Mathematical Practice are the same for all grade levels and are listed below.

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

# Key Academic Terms:

The academic terms included in each instructional support. These academic terms are derived from the standards and are to be incorporated into instruction by the teacher and used by the students.

# Instructional Activities:

A representative set of sample activities and examples that can be used in the classroom. The set of activities and examples is not intended to include all the activities and examples defined by the standard. These will be available in Fall 2020.

# Additional Resources:

Additional resources include resources that are aligned to the standard and may provide additional instructional strategies to help students build toward mastery of the designated standard. These will be available in Fall 2020.

#### The Number System

Know that there are numbers that are not rational, and approximate them by rational numbers.

**8.NS.1** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

# **Guiding Questions with Connections to Mathematical Practices:**

#### How can a number be identified as rational or irrational?

*M.P.2. Reason abstractly and quantitatively.* Identify a number as irrational or rational based on whether it can be written as a fraction. For example,  $\sqrt{5}$  is irrational because it can't be written as a fraction, and 0.375 is rational because it can be written as the fraction  $\frac{3}{8}$ .

# How do the decimal expansions of rational numbers compare to the decimal expansions of irrational numbers?

*M.P.6. Attend to precision.* Recognize that rational numbers either terminate or have repeating digits, and recognize that irrational numbers are non-repeating and non-terminating and so cannot be expressed as fractions. For example, the fraction  $\frac{4}{9}$  can be expressed as a decimal by computing  $4 \div 9$ , which gives  $0.\overline{4}$ . The irrational number  $\sqrt{2}$  written as a decimal is 1.4142... with no repeating pattern or termination, so it cannot be expressed as a fraction unless it is rounded.

# **Key Academic Terms:**

rational, irrational, decimal, fraction, repeating, terminating, approximate, decimal expansion

#### The Number System

Know that there are numbers that are not rational, and approximate them by rational numbers.

**8.NS.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g.,  $\pi^2$ ).

Example: By truncating the decimal expansion of  $\sqrt{2}$ , show that  $\sqrt{2}$  is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

### **Guiding Questions with Connections to Mathematical Practices:**

# How does knowledge of perfect squares of rational numbers help determine the size of irrational numbers?

*M.P.2. Reason abstractly and quantitatively.* Estimate the value of an irrational number by using the closest rational known value or values. For example,  $\sqrt{13}$  can be approximated by first finding the perfect squares 13 is between, which are 9 and 16. Since  $\sqrt{9} = 3$  and  $\sqrt{16} = 4$ ,  $\sqrt{13}$  must be between 3 and 4. Furthermore,  $\sqrt{13}$  must be closer to 4 than to 3 since 13 is closer to 16 than to 9 on a number line.

*M.P.7. Look for and make use of structure.* Estimate the value of an expression by locating the irrational number expression on a number line and using the closest rational value. For example, to approximate  $\pi^2$  on a number line, locate  $\pi$  on a number line, between 3 and 4. That means that  $\pi^2$  will be between 9 and 16 because  $3^2$  is 9 and  $4^2$  is 16. To the nearest tenth,  $\pi$  is between 3.1 and 3.2, which means that  $\pi^2$  is between 9.61 and 10.24. Taking it another step,  $\pi$  is between 3.14 and 3.15, which means that  $\pi^2$  is between 9.8596 and 9.9225. As the pattern continues, the range for the value of  $\pi^2$  gets smaller and becomes a better estimate of the actual value.

# How do rational approximations of irrational numbers help compare the size of irrational numbers?

*M.P.5. Use appropriate tools strategically.* Compare the values of irrational numbers by using a calculator to find approximate numbers in decimal form. For example,  $\sqrt{8} \approx 2.83$  and  $\sqrt{10} \approx 3.16$ . Since 2.83 is less than 3.16,  $\sqrt{8} < \sqrt{10}$ .

# **Key Academic Terms:**

 $rational,\,irrational,\,number\,line,\,approximate,\,estimate$ 

#### **Expressions and Equations**

Work with radicals and integer exponents.

**8.EE.3** Know and apply the properties of integer exponents to generate equivalent numerical expressions.

Example:  $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$ .

### **Guiding Questions with Connections to Mathematical Practices:**

# How can the properties of integer exponents be used to generate equivalent numerical expressions?

*M.P.3.* Construct viable arguments and critique the reasoning of others. Decompose and compose expressions that can be written in the form  $a^m a^n = a^{m+n}$  to create equivalent expressions using exponents. When two terms being multiplied have the same base, the decomposed terms will show that the exponents can be added to find an equivalent expression. For example,  $(-3)^5 \times (-3)^2$  decomposed is  $(-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3)$ , which is equivalent to  $(-3)^7$ . Therefore, to represent  $(-3)^5 \times (-3)^2$ , the exponents of 5 and 2 can be added:  $(-3)^5 \times (-3)^2 = (-3)^{5+2} = (-3)^7$ .

*M.P.3. Construct viable arguments and critique the reasoning of others.* Decompose and compose expressions that can be written in the form  $(a^m)^n = a^{mn}$  to create equivalent expressions using exponents. For example,  $5^6$  decomposed is  $5 \times 5 \times 5 \times 5 \times 5 \times 5$  and can be regrouped as  $(5 \times 5 \times 5) \times (5 \times 5 \times 5)$  or  $5^3 \times 5^3$ , which is equivalent to  $(5^3)^2$ , or  $(5^3)^2 = 5^{3 \cdot 2} = 5^6$ .

*M.P.3. Construct viable arguments and critique the reasoning of others.* Decompose and compose expressions that can be written in the form  $(ab)^n = a^nb^n$  to create equivalent expressions using exponents. When terms are being multiplied together, it is helpful to remember the properties of multiplication. For example,  $(ab)^4$  decomposed is  $(a \times b) \times (a \times b) \times (a \times b) \times (a \times b)$ , which is equivalent to  $(a \times a \times a \times a) \times (b \times b \times b \times b)$  and can also be written as  $a^4b^4$ .

#### How are negative exponents used to represent repeated division?

*M.P.7. Look for and make use of structure.* Connect patterns of exponents to repeated multiplication and division. For example,  $\frac{8^5}{8^3} = \frac{8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8}{8 \cdot 8 \cdot 8} = 8^2$  by decomposition and  $\frac{8^5}{8^3} = 8^{5-3} = 8^2$  using the properties of exponents. The same patterns hold when the power of the denominator is greater than the power of the numerator:  $\frac{8^3}{8^5} = \frac{8 \cdot 8 \cdot 8}{8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8} = \frac{1}{8^2}$  by decomposition and  $\frac{8^3}{8^5} = 8^{3-5} = 8^{-2}$  using the properties of exponents. Therefore,  $8^{-2} = \frac{1}{8^2} = \frac{1}{8 \cdot 8}$  means that a negative exponent can be interpreted to mean repeated division. Another approach is to decompose the expression  $\frac{8^3}{8^5}$  into  $\frac{8 \cdot 8 \cdot 8}{8 \cdot 8 \cdot 8 \cdot 8 \cdot 8}$  and then  $\frac{8}{8} \cdot \frac{8}{8} \cdot \frac{8}{8} \cdot \frac{8}{8} \cdot \frac{1}{8} \cdot \frac{1}{8} = 1 \cdot 1 \cdot 1 \cdot \frac{1}{8} \cdot \frac{1}{8} = \frac{1}{8 \cdot 8} = \frac{1}{64}$ .

#### Why is a nonzero rational number raised to the zero power equivalent to 1?

*M.P.7. Look for and make use of structure.* Demonstrate that as the exponent increases by 1, the base is multiplied by itself one more time, and as the exponent decreases by one, the base is divided by itself one more time. For example,  $\frac{a^2}{a} = (a \cdot a) \cdot \frac{1}{a} = a^1$ . Dividing by a reduced the power by 1, from  $a^2$  to  $a^1$ . Continuing,  $\frac{a}{a} = a \cdot \frac{1}{a} = 1$ . The power reduces by 1, meaning a will be divided by a, so  $a^0 = 1$ . In addition,  $\frac{a}{a} = 1$ , so  $\frac{a}{a} = a^{1-1} = a^0 = 1$ .

# **Key Academic Terms:**

integer, exponent, power, negative, base, reciprocal, inverse, equivalent, radical

#### **Expressions and Equations**

Work with radicals and integer exponents.

**8.EE.4** Use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$  and  $x^3 = p$ , where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that  $\sqrt{2}$  is irrational.

# **Guiding Questions with Connections to Mathematical Practices:**

# What is the relationship between $\sqrt{x}$ and $x^2$ ?

*M.P.2. Reason abstractly and quantitatively.* Define the square root of x as a number that, when multiplied by itself, is equal to x, and x can be decomposed as  $x = \sqrt{x} \cdot \sqrt{x}$ . The exponent of 2 means to use the base as a factor 2 times, so  $x^2$  can be decomposed as  $x^2 = x \cdot x$ . To find the solution to an equation such as  $x^2 = p$ , find a number that when multiplied by itself is p. That number is represented as  $\sqrt{p}$ . The radical symbol in  $\sqrt{p}$  indicates the principle square root, and the answer is the positive solution to  $x^2 = p$ , such as  $\sqrt{16} = 4$ . The square and the square root are inverse operations. For example,  $(\sqrt{25})^2 = 25$  and  $\sqrt{25^2} = 25$ .

# What is the relationship between $\sqrt[3]{x}$ and $x^3$ ?

*M.P.2. Reason abstractly and quantitatively.* Define the cube root of x as a number that, when used as a factor 3 times, has a product that is equal to x, and x can be decomposed as  $x = \sqrt[3]{x} \cdot \sqrt[3]{x}$ . The exponent of 3 means to use the base as a factor 3 times, so  $x^3$  can be decomposed as  $x^3 = x \cdot x \cdot x$ . To find the solution to an equation such as  $x^3 = p$ , find a number that when used as a factor 3 times has a product equal to p. That number is represented as  $\sqrt[3]{p}$ . The cube and the cube root are inverse operations. For example,  $(\sqrt[3]{125})^3 = 125$  and  $\sqrt[3]{125^3} = 125$ .

# **Key Academic Terms:**

rational, square root, cube root, exponent, irrational, power, perfect square, perfect cube, radical, principle square root

#### **Expressions and Equations**

Work with radicals and integer exponents.

**8.EE.5** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

Example: Estimate the population of the United States as  $3 \times 10^8$  and the population of the world as  $7 \times 10^9$ , and determine that the world population is more than 20 times larger.

### **Guiding Question with Connections to Mathematical Practices:**

# How can integer exponents of 10 be used to compare very large and very small numbers?

# **Key Academic Terms:**

integer, power, exponent, significant digit, estimate

#### **Expressions and Equations**

Work with radicals and integer exponents.

**8.EE.6** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

### **Guiding Questions with Connections to Mathematical Practices:**

#### When is it appropriate to use scientific notation?

*M.P.2. Reason abstractly and quantitatively.* Use scientific notation when the context calls for it and the numbers are either very large or very small. For example, use scientific notation as an estimate for the number of stars visible in the sky on a clear night, the number of cells on a microscope slide, or the mass of a planet but not for the amount of change in a pocket or the number of people on a bus.

# How can the properties of exponents be used to solve problems written in scientific notation?

*M.P.6. Attend to precision.* Connect knowledge of properties of exponents to scientific notation to solve problems. For example,  $(4.8 \times 10^{12}) \times (2.1 \times 10^6)$  becomes  $(4.8 \times 2.1) \times (10^{12} \times 10^6)$ , which becomes  $10.08 \times 10^{12+6}$  or  $10.08 \times 10^{18}$ . And  $10.08 \times 10^{18}$  represented in scientific notation is  $1.008 \times 10^{19}$ .

# How is the appropriate unit determined when solving problems using scientific notation?

*M.P.1. Make sense of problems and persevere in solving them.* Determine units by making sense of the context. For example, when calculating large distances, use kilometers instead of centimeters. Or when tracking levels of contaminants in a water system, use milliliters instead of liters.

#### How does technology represent numbers written in scientific notation?

*M.P.5. Use appropriate tools strategically.* Analyze the result of a math problem that requires technology and has a solution that is a very large or very small number. For example, a series of computations on a calculator gives 5.2E31 as an output. Recognize that this is an alternate representation for  $5.2 \times 10^{31}$ .

### **Key Academic Terms:**

scientific notation, significant digit, decimal, exponent, power, unit, radical

#### **Expressions and Equations**

Understand the connections among proportional relationships, lines, and linear equations.

**8.EE.7** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

Example: Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

### **Guiding Questions with Connections to Mathematical Practices:**

How is the unit rate of a proportional relationship found from a table, graph, equation, diagram, or verbal description?

*M.P.8. Look for and express regularity in repeated reasoning.* Find the constant of proportionality to determine the unit rate, which is how much the output value changes when the input value changes by exactly one unit. For example, a table shows how many cars are assembled in a factory for different numbers of days. In 2 days, 400 cars are assembled; in 3 days, 600 cars are assembled, and so on. When the input value increases by 1 day (exactly 1 unit), the output value increases by 200. Therefore, the constant of proportionality is 200 cars per day.

### How does the slope of a graph connect to the unit rate of a proportional relationship?

*M.P.4. Model with mathematics.* Represent the unit rate as the slope of the line in a graph. For example, a line that starts at the origin and passes through the point (2, 7) has a slope of  $\frac{7}{2}$  because the unit rate is  $\frac{7}{2}$ .

# How can two different proportional relationships be compared when represented in different ways?

*M.P.6.* Attend to precision. Compare proportional relationships represented in different ways by finding the unit rate. For example, given one relationship represented by the equation y = x and another relationship represented by a graph of a line with a slope of 3 that goes through the origin, identify that the relationship represented by the graphed line has a rate of change that is 3 times the rate of change of the relationship represented by the equation.

# **Key Academic Terms:**

input, output, variable, equation, proportional, coordinate plane, constant of proportionality, unit rate, slope, graph, table

#### **Expressions and Equations**

Understand the connections among proportional relationships, lines, and linear equations.

**8.EE.8** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.

### **Guiding Questions with Connections to Mathematical Practices:**

#### How can similar triangles show that slope is a constant value on a non-vertical line?

*M.P.3. Construct viable arguments and critique the reasoning of others.* Represent the slope between any two distinct points on a line by drawing similar right triangles to demonstrate that the slope is the same between any two points. The lengths of the vertical legs represent a change in *y*-value between two points on the line, and the lengths of the horizontal legs represent a change in *x*-value between two points on the line. The ratio of the vertical change to the horizontal change is the slope. Because any two of the right triangles are similar, the ratios of the lengths of the vertical and horizontal legs are equivalent; therefore, the slopes are equivalent. For example, a graphed line that goes through the origin and also passes through the points (1, 5), (2, 10), (3, 15), and (4, 20) can have any number of similar right triangles drawn with the line as the hypotenuse, and every triangle on that line will have the ratio of the change in *y* over the change in *x* as a value equivalent to 5.

### How is the equation for a non-vertical line that passes through the origin derived?

*M.P.8. Look for and express regularity in repeated reasoning.* Represent the unit rate as m, so when x changes by one unit, y changes by m units. For example, when x changes by 2, y changes by 2m. Whatever the change in x, the change in y is mx, so y = mx.

#### How is the equation y = mx + b connected to the equation y = mx for graphing lines?

*M.P.7. Look for and make use of structure.* Connect the equation y = mx to y = mx + b by explaining that b represents the y-intercept and, when b = 0, the situation represented is proportional. For example, the equation y = 2x graphs a line through the origin that has a slope of 2, and the equation y = 2x + 3 graphs a line with the same slope that is shifted 3 units higher and crosses the y-axis at 3, making a line that is parallel to the first line.

# **Key Academic Terms:**

similar, right triangle, slope, origin, horizontal, vertical, axis, unit rate, graph, input, output, intercept, coordinate plane, *y*-intercept, hypotenuse, slope triangle

#### **Expressions and Equations**

Analyze and solve linear equations and pairs of simultaneous linear equations.

**8.EE.9** Solve linear equations in one variable.

**8.EE.9a** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).

### **Guiding Questions with Connections to Mathematical Practices:**

#### How are the properties of operations used to solve equations in one variable?

*M.P.6. Attend to precision.* Use the properties of operations, such as the subtraction property of equality, and the structure of problems to solve linear equations in one variable. For example, 3a + 44 = 194 can be rewritten as 3a = 150, which means that a = 50 makes the equation true; therefore, 50 is a solution to 3a + 44 = 194.

# How can it be determined when an equation with one variable has one solution, infinitely many solutions, or no solutions?

*M.P.2. Reason abstractly and quantitatively.* Find an equivalent equation in the form of x = a, a = a, or a = b to determine whether a linear equation has one solution, infinitely many solutions, or no solutions. For example, t + 2 = 2 + t has infinitely many solutions because any value of t yields the result 2 = 2, and the equation h + 3 = h - 4 has no solutions because any value of t yields the result t = -4, which is not true, so there is no value of t for which the equation is true.

# **Key Academic Terms:**

equation, variable, linear, equivalent, constant, solution, infinite, simultaneous linear equations

#### **Expressions and Equations**

Analyze and solve linear equations and pairs of simultaneous linear equations.

**8.EE.9** Solve linear equations in one variable.

**8.EE.9b** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions, using the distributive property and collecting like terms.

### **Guiding Question with Connections to Mathematical Practices:**

#### How can equations in one variable with rational coefficients be solved?

*M.P.6. Attend to precision.* Use the properties of operations, the structure of the problem, and/or equivalent expressions to solve for the unknown quantity in an equation. For example, use the properties of operations to solve  $\frac{4}{3}c + 7 = \frac{2}{3}c - 1$ . First, add the opposites of 7 and  $\frac{2}{3}c$  to collect like terms on both sides of the equal sign, resulting in  $\frac{4}{3}c - \frac{2}{3}c = -1 - 7$ . Combining like terms gives the equivalent equation  $\frac{2}{3}c = -8$ . Multiplying both sides of the equation by  $\frac{3}{2}$  gives the equivalent equation  $\frac{3}{2} \cdot \frac{2}{3} \cdot c = \frac{3}{2}(-8)$ . Therefore, c = -12.

# **Key Academic Terms:**

equation, variable, linear, equivalent, constant, solution, like terms, distributive property, coefficient, reciprocal, unknown quantity, expressions

#### **Expressions and Equations**

Analyze and solve linear equations and pairs of simultaneous linear equations.

**8.EE.10** Analyze and solve pairs of simultaneous linear equations.

**8.EE.10a** Understand that solutions to a system of two linear equations in two variables correspond to points of intersections of their graphs because points of intersection satisfy both equations simultaneously.

### **Guiding Question with Connections to Mathematical Practices:**

# How does the point where two lines intersect on a graph help solve a system of equations?

*M.P.4. Model with mathematics.* Observe that the solution to a system of equations is the intersection of the lines of the two equations at the coordinates (x, y). For example, the graphs of y = 3x and y = 5x - 2 intersect at the point (1, 3), so the solution to the system of equations is x = 1 and y = 3 because those values are solutions to both equations, as 3 = 3(1) and 3 = 5(1) - 2.

*M.P.4. Model with mathematics.* Observe that, when two equations have the same graph, the solution to the system of equations is every point on the line. For example, the graphs of y = 1.5x + 8 and  $y = \frac{3}{2}x + 8$  intersect at every value of x and y, so every point on the graphs is a solution to both equations.

# **Key Academic Terms:**

linear equation, intersection, graph, variable, system of equations, solution, coordinate

#### **Expressions and Equations**

Analyze and solve linear equations and pairs of simultaneous linear equations.

**8.EE.10** Analyze and solve pairs of simultaneous linear equations.

**8.EE.10b** Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

Example: 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.

### **Guiding Questions with Connections to Mathematical Practices:**

How can the structure of the equations in a system of equations be used to determine a strategy for finding a solution?

*M.P.1. Make sense of problems and persevere in solving them.* Analyze the structure of the given linear equations to plan a solution pathway. For example, the structure of the equations y = 5 - 3x and 2x + 2y = 7 makes substitution a clear method for solving the system because one of the equations is already solved for one of the variables.

#### How do the graphs of two linear equations help estimate the solution?

*M.P.4. Model with mathematics.* Use a graph to estimate solutions to systems of linear equations. For example, a graph with two parallel lines has no solution.

# **Key Academic Terms:**

linear equation, intersection, graph, variable, system of equations, substitution, equivalent, elimination, solution, algebraic, parallel lines

#### **Expressions and Equations**

Analyze and solve linear equations and pairs of simultaneous linear equations.

**8.EE.10** Analyze and solve pairs of simultaneous linear equations.

**8.EE.10c** Solve real-world and mathematical problems leading to two linear equations in two variables.

Example: Given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

#### **Guiding Question with Connections to Mathematical Practices:**

#### How do systems of linear equations help solve real-world mathematical problems?

*M.P.4. Model with mathematics.* Represent and solve situations with two linear equations in two variables. For example, represent the situation "Fred is four times as old as his sister. In 12 years, he will be 4 less than twice her age" with the equation 4s = f for the present day and 2(s+12) - 4 = f + 12 for 12 years from now. This system of equations can be solved algebraically by substituting the 4s for f in the equation that represents 12 years from now. With the substitution, the equation becomes 2(s+12) - 4 = 4s + 12. Using the distributive property, the next step to solving is 2s + 24 - 4 = 4s + 12, and then combining like terms gives 2s + 20 = 4s + 12. The subtraction property of equality is next, which gives 2s = 8, and then the division property of equality gives s = 4. In the context of this situation, s is the age of Fred's sister in the present day, so that means his sister is 4 years old. Use s = 4 in the equation 4s = f to find Fred's age:  $4 \cdot 4 = f$ , or 16 = f. Fred is 16 years old.

### **Key Academic Terms:**

linear equation, intersection, graph, variable, system of equations, substitution, equivalent, elimination

#### **Functions**

Define, evaluate, and compare functions.

**8.F.11** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)

# **Guiding Questions with Connections to Mathematical Practices:**

#### How do patterns, rules, and proportional relationships connect to functions?

*M.P.2. Reason abstractly and quantitatively.* Observe functions in terms of a pattern, rule, or proportional relationship to make sense of how the variables relate to each other. For example, skip-counting by 15 is a pattern that can be represented by y = 15x, where x is the number of each term in the pattern, y is the value of that term, and the rule is to skip-count by 15.

#### How can it be determined whether a table or a graph represents a function?

*M.P.6.* Attend to precision. Observe the table or graph carefully and note whether each input corresponds to only one output. For example, a table might have each input listed only once, or if it is listed more than once, the same output corresponds to it each time.

# **Key Academic Terms:**

input, output, function, graph, table, pattern, rule, proportional relationship, ordered pairs

#### **Functions**

Define, evaluate, and compare functions.

**8.F.12** Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Example: Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

### **Guiding Question with Connections to Mathematical Practices:**

#### How are different representations of the same function connected?

*M.P.1. Make sense of problems and persevere in solving them.* Determine and explain correspondences between equations, verbal descriptions, tables, and graphs. For example, the initial value, the output when the input is zero, can be found in any of the forms and is helpful when comparing functions.

# **Key Academic Terms:**

input, output, function, graph, table, rate of change, initial value, verbal description, slope

#### **Functions**

Define, evaluate, and compare functions.

**8.F.13** Interpret the equation y = mx + b as defining a linear function whose graph is a straight line; give examples of functions that are not linear.

Example: The function  $A = s^2$  giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4), and (3, 9), which are not on a straight line.

### **Guiding Questions with Connections to Mathematical Practices:**

#### How can it be determined that the graph of y = mx + b is a line?

*M.P.4. Model with mathematics.* Demonstrate that y = mx + b is a linear function whose graph is a line by showing that the slope between any two pairs of points on the line is the same. For example, y = 2x - 4 is a function that is a line when graphed on a coordinate plane because the slope between (1, -2) and (2, 0) is 2, the slope between (3, 2) and (4, 4) is also 2, and the slope between any two pairs of points of the function will always be 2.

#### How can it be determined whether a function is linear or not linear?

*M.P.6. Attend to precision.* Determine that nonlinear functions have a rate of change that varies between different pairs of points, which means their graphs are not lines. For example, the table of a function has the points (0, 0), (1, 1), (2, 4), and (3, 9). The rate of change between (0, 0) and (1, 1) is 1. The rate of change between (2, 4) and (3, 9) is 5. The rate of change is not the same between the pairs of points, so the function is nonlinear.

# **Key Academic Terms:**

input, output, function, linear, rate of change, graph, nonlinear, side length, linear function, nonlinear function

#### **Functions**

Use functions to model relationships between quantities.

**8.F.14** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of linear function in terms of the situation it models and in terms of its graph or a table of values.

### **Guiding Questions with Connections to Mathematical Practices:**

# How can a function be constructed to model a linear relationship between two quantities?

*M.P.4. Model with mathematics.* Look for the features of a function, such as rate of change (m) and the point (0, b), also called the y-intercept, within a relationship. For example, the input is always the independent variable and the output is always the dependent variable. The rate of change is the change in the dependent variable (y) divided by the change in the corresponding independent variable (x). The function is constructed by using the rate of change, initial value, and variables to show the relationship between the quantities.

# How can the rate of change and initial value be determined from a description of a relationship, two (x, y) values, a graph, or a table?

*M.P.4. Model with mathematics.* Make use of what is known about the relationship and the meanings of x and y in a function. For example, a line that passes through the points (3, -4) and (5, 2) changes by 6 in the y-value and by 2 in the x-value, making the ratio of the y-value to the x-value  $\frac{6}{2} = \frac{3}{1}$ . Then use one of the given points and the rate of change, m = 3, to solve for the initial value, b, in y = mx + b.

#### What do the rate of change and initial value represent in a given context?

*M.P.2. Reason abstractly and quantitatively.* Interpret the meaning of the rate of change and initial value for a context and the table or graph of that context. For example, a scenario where someone earns \$15 per week in allowance and starts with \$50 in savings would result in an initial value of \$50 and a rate of change of \$15 per week, so the graph would start at (0, 50) and increase by 15 for each weekly increment.

# **Key Academic Terms:**

input, output, function, linear, initial value, rate of change, independent variable, dependent variable, y-intercept

#### **Functions**

Use functions to model relationships between quantities.

**8.F.15** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

### **Guiding Questions with Connections to Mathematical Practices:**

# How can analyzing a graph qualitatively help determine the functional relationship between two quantities?

*M.P.1. Make sense of problems and persevere in solving them.* Analyze a graph using its features and general shape, reading from left to right to describe what happens to the output as the input increases. For example, an increasing linear graph comparing time to distance shows that the rate of change is the same for every interval, so the speed is constant.

# How can the qualitative features of the verbal descriptions of a function be represented in a graph?

*M.P.4. Model with mathematics.* Represent a function with a sketched graph to show the relationship between the two quantities. For example, to show a bank account that starts with a large amount of money and decreases by varying small amounts of money each month for fees and withdrawals, use a decreasing nonlinear graph.

# **Key Academic Terms:**

input, output, function, linear, nonlinear, interval, rate of change, constant speed, qualitative, sketch

#### Geometry

Understand congruence and similarity using physical models, transparencies, or geometry software.

- **8.G.16** Verify experimentally the properties of rotations, reflections, and translations:
- **8.G.16a** Lines are taken to lines, and line segments are taken to line segments of the same length.

### **Guiding Questions with Connections to Mathematical Practices:**

#### What strategies can be used to understand rigid motions?

*M.P.5.* Use appropriate tools strategically. Demonstrate rigid transformations on a variety of figures using a variety of tools. For example, place a piece of tracing paper over a shape on a piece of paper and trace the shape. Then slide the tracing paper to a new location to demonstrate a translation.

# What effects do rotations, reflections, and translations have on lines and line segments?

*M.P.4. Model with mathematics.* Observe rotations, reflections, and translations of lines and line segments to conclude that the size of segments stays the same for these transformations and that both lines and line segments will be oriented differently but will remain lines and line segments of the same length. For example, the line y = 5x + 1 rotated 90 degrees clockwise about the origin will remain a line, but the image will have a negative slope and an x-intercept of 1, instead of a positive slope and a y-intercept of 1 like the pre-image.

# **Key Academic Terms:**

rotation, reflection, translation, transformation, rigid motion, image, pre-image, line, line segment, center of rotation, line of reflection, clockwise, counterclockwise

#### Geometry

Understand congruence and similarity using physical models, transparencies, or geometry software.

- **8.G.16** Verify experimentally the properties of rotations, reflections, and translations:
- **8.G.16b** Angles are taken to angles of the same measure.

### **Guiding Question with Connections to Mathematical Practices:**

#### What effects do rotations, reflections, and translations have on angle measures?

*M.P.4. Model with mathematics.* Observe rotations, reflections, and translations of angles to conclude that the measurement of the angles does not change with rigid motions. For example, a triangle that is reflected across the *y*-axis in a coordinate plane will have the same angle measures as the pre-image.

# **Key Academic Terms:**

rotation, reflection, translation, transformation, rigid motion, image, pre-image, line, line segment, center of rotation, line of reflection, angle

#### Geometry

Understand congruence and similarity using physical models, transparencies, or geometry software.

- **8.G.16** Verify experimentally the properties of rotations, reflections, and translations:
- **8.G.16c** Parallel lines are taken to parallel lines.

### **Guiding Question with Connections to Mathematical Practices:**

#### What effects do rotations, reflections, and translations have on parallel lines?

*M.P.4. Model with mathematics.* Observe rotations, reflections, and translations of parallel lines to conclude that the lines remain parallel with rigid motions. For example, parallel lines that are translated 3 units to the left and then 2 units down on a coordinate plane remain parallel lines with the same space between them.

# **Key Academic Terms:**

rotation, reflection, translation, transformation, rigid motion, image, pre-image, line, line segment, center of rotation, line of reflection, parallel

#### Geometry

Understand congruence and similarity using physical models, transparencies, or geometry software.

**8.G.17** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

# **Guiding Questions with Connections to Mathematical Practices:**

# How can it be determined that a two-dimensional figure is congruent to another two-dimensional figure?

*M.P.6.* Attend to precision. Show that one figure is congruent to another using rigid motions. For example, demonstrate that two figures are congruent by showing that one figure is the pre-image and one is the same figure after being reflected across a line and then rotated 90° clockwise.

# How can a strategy be determined to describe a sequence of transformations that shows congruency between figures?

*M.P.1. Make sense of problems and persevere in solving them.* Analyze two figures to determine a sequence of transformations that show congruency. For example, when using a mirror, geometry software, or paper folding, if a figure and its pre-image are determined to be mirror images of each other, reflection must be included in the sequence of transformations because mirror images maintain congruent angle measures and side lengths.

# **Key Academic Terms:**

sequence, rotation, reflection, translation, transformation, rigid motions, image, pre-image, congruent, mirror image

#### Geometry

Understand congruence and similarity using physical models, transparencies, or geometry software.

**8.G.18** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

## **Guiding Questions with Connections to Mathematical Practices:**

#### How are dilations different from other types of transformations?

*M.P.2. Reason abstractly and quantitatively.* Describe the transformation of dilation as making an image that is similar to the original image (pre-image), which means it will be the same shape but a different size. Dilations change the size of the pre-image, whereas translations, rotations, and reflections do not change the size of the image. For example, a rectangle that is dilated by a factor of 3 will still be a rectangle with four right angles, but the side lengths for the image will be three times the length of the corresponding sides of the pre-image.

# How can the coordinates of a figure be determined when a dilation is performed on the pre-image of the figure?

*M.P.7. Look for and make use of structure.* Identify the coordinates of the pre-image, and use the description of the dilation to find the coordinates of the image. For example, to find the coordinates of an image of a square with the coordinates (2, 2), (6, 2), (6, 6), and (2, 6) under a dilation of  $\frac{1}{2}$  with a center of dilation at the origin, start with the coordinates of the pre-image square and multiply each coordinate by  $\frac{1}{2}$ . The image of the square will have the coordinates (1, 1), (3, 1), (3, 3), and (1, 3).

# How can the coordinates of a figure be determined when a rigid motion is performed on the pre-image of the figure?

*M.P.7. Look for and make use of structure.* Identify the coordinates of the pre-image and use the description of the translations, rotations, and/or reflections to find the coordinates of the new figure. For example, to find the new coordinates of a triangle that is translated 4 units to the left, subtract 4 from each x-coordinate. So a triangle with vertices at (1, 2), (3, 1), and (4, 4) translated 4 units to the left will have the new coordinates (-3, 2), (-1, 1), and (0, 4).

# **Key Academic Terms:**

rotation, reflection, translation, transformation, rigid motion, image, pre-image, congruent, similar, preserve, coordinates, dilation, vertex

#### Geometry

Understand congruence and similarity using physical models, transparencies, or geometry software.

**8.G.19** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

### **Guiding Questions with Connections to Mathematical Practices:**

# How can it be determined that a two-dimensional figure is similar to another two-dimensional figure?

*M.P.8. Look for and express regularity in repeated reasoning.* Examine the figures' angles and side lengths to determine similarity. For example, a shape that is reflected across a line, rotated 90 degrees counterclockwise and dilated by a factor of 4 will have equal angle measures to the pre-image and side lengths that are all 4 times longer than the pre-image, making it similar to the pre-image.

# How can the sequence of transformations that shows similarity between figures be determined?

*M.P.1. Make sense of problems and persevere in solving them.* Look for a sequence of transformations from one figure to the next. For example, if the image has line segments that are 10 units long and the pre-image has a corresponding line segment that is 2 units long, the dilation was by a factor of 5. If the image and pre-image have corresponding angles of equal measure and the corresponding sides are flipped, then the sequence of transformations includes a reflection.

# **Key Academic Terms:**

rotation, reflection, translation, transformation, rigid, image, pre-image, congruent, preserve, dilation, similar, corresponding sides, corresponding angles, exhibit

#### Geometry

Understand congruence and similarity using physical models, transparencies, or geometry software

**8.G.20** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Example: Arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give argument in terms of transversals why this is so.

### **Guiding Questions with Connections to Mathematical Practices:**

#### Why is the sum of the angles of a triangle 180°?

 $\it M.P.7. Look$  for and make use of structure. Demonstrate that the angle measures of triangles always sum up to  $180^{\circ}$  in a variety of ways. For example, show that a triangle with angle measures of  $80^{\circ}$ ,  $40^{\circ}$ , and  $60^{\circ}$  has the angle sum of  $180^{\circ}$  by cutting the angles out and placing them together to show that they make a line.

#### How are the angles created by a transversal through parallel lines related?

*M.P.3. Construct viable arguments and critique the reasoning of others.* Explain the angles of equal measure and the supplementary angles formed with a transversal through parallel lines. Use a transversal to informally prove whether lines are parallel. For example, use rotations to show that alternate interior angles have equal measure.

#### How can the angle-angle criterion be used to determine triangle similarity?

*M.P.6. Attend to precision.* Observe that triangles with equal corresponding angle measures are similar, and if two pairs of corresponding angles are known to be congruent, then triangle properties, such as the triangle sum theorem, can be used to prove the third angles are congruent. For example, any triangle with two angles measuring  $30^{\circ}$  and  $45^{\circ}$  must have a third angle of  $180^{\circ} - (30^{\circ} + 45^{\circ}) = 105^{\circ}$ .

# **Key Academic Terms:**

triangle, angle measure, parallel lines, transversal, angle-angle criterion, similarity, corresponding, alternate interior angles, alternate exterior angles, supplementary angles, vertical angles, triangle sum theorem

#### Geometry

Understand and apply the Pythagorean Theorem.

**8.G.21** Explain a proof of the Pythagorean Theorem and its converse.

#### **Guiding Questions with Connections to Mathematical Practices:**

# How can prior knowledge of triangles and squares be used to analyze and justify the Pythagorean Theorem?

*M.P.4. Model with mathematics.* Use the side lengths and areas of squares to demonstrate the Pythagorean Theorem with corresponding right triangles. For example, create squares with the same side lengths as each side of a right triangle, and use the areas of the squares to form ideas about the relationship between the legs and hypotenuse of the right triangle.

# How can the converse of the Pythagorean Theorem be used to determine the type of triangle, given its side lengths?

*M.P.7. Look for and make use of structure.* Use the square of the longest side of a triangle and the sum of the squares of the two other sides to determine whether the triangle is a right, obtuse, or acute triangle. For example, a triangle with side lengths of 10, 24, and 26 must be a right triangle because  $26^2 = 10^2 + 24^2$ . By contrast, a triangle with side lengths of 3, 5, and 6 is not a right triangle, because  $6^2 > 3^2 + 5^2$ , which means the triangle is obtuse, and a triangle with side lengths of 5, 9, and 10 is acute because  $10^2 < 5^2 + 9^2$ .

# **Key Academic Terms:**

right triangle, Pythagorean Theorem, leg, hypotenuse, converse, sides, obtuse triangle, acute triangle, proof

#### Geometry

Understand and apply the Pythagorean Theorem.

**8.G.22** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

### **Guiding Question with Connections to Mathematical Practices:**

# How can the Pythagorean Theorem be used to find unknown side lengths in right triangles?

*M.P.6. Attend to precision.* Substitute the known side lengths into the equation  $a^2 + b^2 = c^2$  and then solve the equation for the unknown side length in two and three dimensions. For example, if the structure underneath a walking bridge is made up of right triangles that each have a hypotenuse that is 12 feet long and one leg that is 9 feet long, find the length of the missing leg by solving the equation  $a^2 + 9^2 = 12^2$  for a.

### **Key Academic Terms:**

right triangle, Pythagorean Theorem, leg, hypotenuse, dimensions, unit

#### Geometry

Understand and apply the Pythagorean Theorem.

**8.G.23** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

### **Guiding Question with Connections to Mathematical Practices:**

# How can the Pythagorean Theorem be used to find the distance between points in a coordinate system?

*M.P.7. Look for and make use of structure.* Find the right triangle that corresponds to any two points on a coordinate grid by drawing a line segment between them, and then determine the horizontal and vertical legs from the two points that meet at a right angle; the distance between the two points is the hypotenuse of a right triangle. For example, the points (1, 1) and (5, 6) make the hypotenuse, c, of a triangle that has its right angle vertex at (5, 1). As such, the right triangle has legs of length 4 and 5, and so the distance between (1, 1) and (5, 6) will be the value that solves the equation  $c^2 = 4^2 + 5^2$ .

### **Key Academic Terms:**

right triangle, Pythagorean Theorem, legs, hypotenuse, distance, coordinate system, line segment, vertex

#### Geometry

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

**8.G.24** Know the formulas for the volumes of cones, cylinders, and spheres, and use them to solve real-world and mathematical problems.

# **Guiding Questions with Connections to Mathematical Practices:**

#### How are the volumes of cones and cylinders connected to the areas of circles?

*M.P.7. Look for and make use of structure.* Demonstrate the volume of a cone, cylinder, or sphere as the area of a circle, a plane section of the figure, multiplied by another dimension of the figure. For example, recognize the formula for the volume of a cylinder as the area of a circular base multiplied by the height.

#### How are the volumes of cones and cylinders connected to each other?

*M.P.7. Look for and make use of structure.* Explore volume with manipulatives and formulas to find the pattern of multiplying the area of a circle by the constant appropriate to the given figure. For example, a cone has the volume of the corresponding cylinder (i.e., with the same base and height) multiplied by one-third.

# How can real-world and mathematical problems be solved using the volumes of cones, cylinders, and spheres?

*M.P.6.* Attend to precision. Represent the unknown in a problem involving the volume of a cone, cylinder, or sphere with a variable and use the formula to solve for the unknown. For example, given the height and radius of a cylinder-shaped food container, use the formula to find the volume of food the container can hold.

# **Key Academic Terms:**

volume, cone, cylinder, sphere, pi, constant, radius, diameter

#### **Statistics and Probability**

Investigate patterns of association in bivariate data.

**8.SP.25** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

### **Guiding Questions with Connections to Mathematical Practices:**

#### How are graphs for bivariate data constructed?

*M.P.4. Model with mathematics.* Interpret the two quantities as the *x*- and *y*-variables on a coordinate grid to construct a scatter plot. For example, the daily high temperatures during the month of August can be represented with *x*-values while the number of frozen desserts sold at a concession stand each day in August can be represented with *y*-values.

#### How do scatter plots show the relationship between two quantities?

*M.P.2. Reason abstractly and quantitatively.* Observe a scatter plot to determine any patterns that occur and interpret the meaning of those patterns within the given context. For example, a scatter plot that shows Julia's age in years on the *x*-axis and Julia's height in inches on the *y*-axis will reveal a pattern that as Julia gets older, she generally grows taller, up to a certain age, and then her height remains constant.

#### How can scatter plots of bivariate data be described?

*M.P.6.* Attend to precision. Represent the patterns in a scatter plot with terms such as clustering, outliers, positive or negative association, linear association, and nonlinear association. For example, a scatter plot can be described as having a positive linear association or a negative linear association, and may also have some clustering and outliers, but the scatter plot will not have both a positive association and negative association.

# **Key Academic Terms:**

scatter plot, bivariate, data, clustering, positive association, negative association, outlier, linear association, nonlinear association, univariate

#### **Statistics and Probability**

Investigate patterns of association in bivariate data.

**8.SP.26** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

# **Guiding Questions with Connections to Mathematical Practices:**

# When can a relationship between two quantitative variables on a scatter plot be modeled by a straight line?

*M.P.4. Model with mathematics.* Represent a scatter plot with a line of best fit when there is a clear linear association between the two variables. For example, a scatter plot with a negative linear association would have a line with a negative slope that comes close to most of the data points on the graph.

# How can a line be graphed on a scatter plot to best show the relationship between two variables?

*M.P.8. Look for and express regularity in repeated reasoning.* Find a line with an approximate slope and *y*-intercept to fit the scatter plot data points. For example, lay a string on top of a scatter plot and keep the string as close to all the data points as possible to visualize a line that fits the data. Use the approximate slope and *y*-intercept to write a corresponding equation.

*M.P.6. Attend to precision.* Analyze the effect that outliers have when fitting a line to data and adjust the line according to the outlier. For example, notice that an extreme outlier of a data set will have a more significant effect on the fit of a line than an outlier that is less extreme.

# **Key Academic Terms:**

scatter plot, bivariate, data, clustering, association, outlier, line of best fit, quantitative

#### **Statistics and Probability**

Investigate patterns of association in bivariate data.

**8.SP.27** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

Example: In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

### **Guiding Question with Connections to Mathematical Practices:**

# How can the equation of a linear model help solve problems in the context of bivariate data?

*M.P.4. Model with mathematics.* Interpret the slope and intercepts of a fitted line in context to better understand bivariate data and solve problems. For example, if a scatter plot with fitted line y = -8x + 29 models the amount of water, in milliliters, in a container after x days, then the data have a decreasing association because the slope of the line is -8, which means that the amount of water is decreasing by approximately 8 milliliters per day. The container started with 29 milliliters of water, so the initial value is 29 because when x = 0, the y value is 29.

# **Key Academic Terms:**

scatter plot, bivariate, data, clustering, increasing association, decreasing association

#### **Statistics and Probability**

Investigate patterns of association in bivariate data.

**8.SP.28** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Example: Collect data from students in your class on whether or not they have a curfew on school nights, and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

### **Guiding Questions with Connections to Mathematical Practices:**

#### How can two-way tables for bivariate data be constructed?

*M.P.4. Model with mathematics.* Organize the data in a two-way table so that each cell in the table represents the frequency count or a proportion of the two categories. For example, in a survey of seventh- and eighth-grade students and their preference for wearing a hat or not, the two rows will be the grades, and the two columns will be "Wearing a Hat" and "Not Wearing a Hat." As such, each cell will show the number of students in the given grade who have the given preference.

#### How is a two-way table using frequencies related to a two-way table using proportions?

*M.P.7. Look for and make use of structure.* Describe the relationships between the different data representations in two-way tables and determine how to create one type of two-way table given a different type of two-way table. For example, using a two-way table with percentages, determine the frequencies for each part of the table by finding the percentage of the total number of data points for each cell in the table. If 13% of eighth-grade students bring their lunch to school each day, find the frequency of that data point by finding 13% of the total number of eighth-grade students to represent the information of 13% in a different way.

# How can data in a two-way table be analyzed and interpreted to determine any associations between variables?

*M.P.2. Reason abstractly and quantitatively.* Analyze a two-way table to determine any patterns that occur. For example, a survey of students in an elementary school who walk or ride the bus to school compared to students in a high school who walk or ride the bus to school might have patterns of association that show that high school students are more likely to walk to school than elementary school students (or vice versa).

### **Key Academic Terms:**

bivariate, data, categorical, frequency, two-way table, relative frequency, cell, column, row