ACAP

Alabama Comprehensive Assessment Program

SUMMATIVE

Grade 3 Mathematics

Alabama Educator Instructional Supports

Alabama Course of Study Standards

DATA RECOGNITION DRC CORPORATION

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Introduction

The *Alabama Course of Study Instructional Supports: Math* is a companion manual to the 2016 *Revised Alabama Course of Study: Math* for Grades K–12. Instructional supports are foundational tools teachers may use to help students become independent learners as they build toward mastery of the *Alabama Course of Study* content standards.

- The purpose of the instructional supports found in this manual is to help teachers engage their students in exploring, explaining, and expanding their understanding of the content standards.

The content standards contained within the course of study may be accessed on the Alabama State Department of Education (ALSDE) website at www.alsde.edu.

Educators are reminded that content standards indicate minimum content—what all students should know and be able to do by the end of each grade level or course. Local school systems may have additional instructional or achievement expectations and may provide instructional guidelines that address content sequence, review, and remediation.

Organization

The organizational components of this manual include standards, guiding questions, connections to instructional supports, key academic terms, and examples of activities. The definition of each component is provided below:

Content Standard:	The content standard is the statement that defines what all students should know and be able to do at the conclusion of a given grade level or course. Content Standards contain minimum required content and complete the phrase "Students will."
Guiding Questions:	Each guiding question is designed to create a framework for the given standard. Therefore, each question is written to help teachers convey important concepts within the standard. By utilizing guiding questions, teachers are engaging students in investigating, analyzing, and demonstrating knowledge of the underlying concepts reflected in the standard.

Connection to Instructional Supports: The purpose of each instructional support is to engage students in exploring, explaining, and expanding their understanding of the content standards provided in the 2016 *Revised Alabama Course of Study: Math*. An emphasis is placed on the integration of the eight Standards for Mathematical Practice.

Mathematical Practices

The Standards for Mathematical Practice describe the varieties of expertise that mathematics educators at all levels should seek to develop in their students. They rest on the National Council of Teachers of Mathematics (NCTM) process standards and the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up: Helping Children Learn Mathematics.*

The Standards for Mathematical Practice are the same for all grade levels and are listed below.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Key Academic Terms: The academic terms included in each instructional support. These academic terms are derived from the standards and are to be incorporated into instruction by the teacher and used by the students.

Instructional Activities: A representative set of sample activities and examples that can be used in the classroom. The set of activities and examples is not intended to include all the activities and examples defined by the standard. These will be available in Fall 2020.

Additional Resources: Additional resources include resources that are aligned to the standard and may provide additional instructional strategies to help students build toward mastery of the designated standard. These will be available in Fall 2020.

Operations and Algebraic Thinking
Represent and solve problems involving multiplication and division.
3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. Example: Describe a context in which a total number of objects can be expressed as 5 × 7.

Guiding Questions with Connections to Mathematical Practices:

How can multiplication problems be interpreted?

M.P.7. Look for and make use of structure. Understand that a multiplication problem can be interpreted as x groups of y objects; skip-counting by x a total of y times or skip-counting by y a total of x times; or an array with x rows and y columns or y rows and x columns. For example, 4×6 can be interpreted as 4 groups of 6 blocks or as 6 groups of 4 blocks.

How can a multiplication problem be represented visually?

M.P.1. Make sense of problems and persevere in solving them. Create a variety of visual models to represent a multiplication problem and solve by counting. For example, to solve the problem "Amy has 4 bags with 6 marbles in each bag. How many marbles does Amy have altogether?", the problem can be modeled by creating an array of dots in 4 rows of 6 and skip-counting by 6 to represent the 4 rows, resulting in 24 marbles.

Key Academic Terms:

product, groups of equal size, array, row, column, skip-count

Operations and Algebraic Thinking
Represent and solve problems involving multiplication and division.
3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. Example: Describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

Guiding Questions with Connections to Mathematical Practices:

How can real-world situations be modeled by division problems?

M.P.4. Model with mathematics. Understand that a division expression represents either the number of objects in each group when the total number is partitioned evenly into a given number of groups or the number of groups when the total number is partitioned into groups that each contain a given number. For example, $42 \div 7$ models the number of berries in each bowl when 42 berries are divided into 7 bowls or the number of bowls when 42 berries are divided by placing 7 berries in each bowl.

How can multiplication facts be used to solve division problems?

M.P.7. Look for and make use of structure. Use the relationships between multiplication and division to solve problems. For example, the result of the division expression $42 \div 7$ can be found by relating it to the multiplication equation $7 \times 6 = 42$.

Key Academic Terms:

quotient, partition, division, multiplication, expression, equal share

Operations and Algebraic Thinking
Represent and solve problems involving multiplication and division.
3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Guiding Questions with Connections to Mathematical Practices:

How can a word problem be interpreted to determine the unknown in a multiplication or division problem?

M.P.7. Look for and make use of structure. Understand that a word problem with an unknown product is a multiplication problem, and a word problem with an unknown number of groups or an unknown group size can be thought of as both a multiplication problem with unknown factors and as a division problem. For example, the problem "There are 15 candies that are placed in jars, with 5 in each jar. How many jars are needed?" can be represented as $15 \div 5 = \square$, $5 \times \square = 15$, or $\square \times 5 = 15$.

How can a real-world multiplication or division problem be represented in a variety of ways?

M.P.5. Use appropriate tools strategically. Represent real-world multiplication and division problems in a variety of ways. For example, to model $4 \times 8 = \square$, make 4 groups containing 8 objects, and make an array by drawing 4 rows of dots with 8 dots in each row.

Key Academic Terms:

equal groups, array, factor, product, multiplication, division, unknown, equation, represent, measurement quantities, row, column

Operations and Algebraic Thinking
Represent and solve problems involving multiplication and division.
3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. Example: Determine the unknown number that makes the equation true in each of the equations, 8 × ? = 48, 5 = □ ÷ 3, and 6 × 6 = ?.

Guiding Question with Connections to Mathematical Practices:

How can an unknown number in a multiplication or division problem be found?

M.P.7. Look for and make use of structure. Understand that the unknown number in a multiplication or division equation is the number that makes the equation true. Use the meanings of multiplication and division and the relationship between multiplication and division to determine the unknown number. For example, $4 = 12 \div \square$ is the same as $4 \times \square = 12$. Both equations model the question "Given 4 groups, how many are in each group if the total is 12?" and can be used to determine that 3 groups of 4 are equal to 12.

Key Academic Terms:

unknown, multiplication, division, equation, product

Operations and Algebraic Thinking
Understand properties of multiplication and the relationship between multiplication and division.
3.OA.5 Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property)

Guiding Questions with Connections to Mathematical Practices:

How does grouping numbers to find a known fact help solve multiplication and division problems?

M.P.1. Make sense of problems and persevere in solving them. Group known facts in an expression. For example, $6 \times 2 \times 5$ can be reorganized using properties of operations as $6 \times (2 \times 5)$, or 6×10. Also, knowing that $9 \times 4 = 36$ means knowing that $36 \div 4 = 9$ and $36 \div 9 = 4$.

How can numbers be decomposed to solve multiplication and division problems?

M.P.1. Make sense of problems and persevere in solving them. Decompose a multiplication expression into the sum of two multiplication expressions. For example, 9×6 can be decomposed to $9 \times (5 + 1)$, then rewritten as $(9 \times 5) + (9 \times 1)$, which is $45 + 9$. Also, $32 \div 4$ can be decomposed into 32 divided by 2 and then divided by 2 again, or $32 \div 2 = 16$, and then $16 \div 2 = 8$.

How can the properties of operations be shown using visual representations?

M.P.4. Model with mathematics. Represent how to solve multiplication and division problems using the properties of operations with models. For example, use an array or area model to show that 7×4 is the same as $(7 \times 2) + (7 \times 2)$.

Why does reordering numbers to find a known fact help solve multiplication problems but not division problems?

M.P.3. Construct viable arguments and critique the reasoning of others. Understand that knowing the result of a multiplication expression $a \times b$ also means knowing the result of $b \times a$. Recognize that this strategy works for multiplication, but not division. For example, knowing $7 \times 3 = 21$ means also knowing $3 \times 7 = 21$. However, $21 \div 3$ is not the same as $3 \div 21$.

Key Academic Terms:

product, sum, property of operations, multiplication expression, decompose, array, area model

Operations and Algebraic Thinking
Understand properties of multiplication and the relationship between multiplication and division.
3.OA.6 Understand division as an unknown-factor problem. Example: Find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

Guiding Question with Connections to Mathematical Practices:

Why can a division problem be thought of as an unknown-factor problem?

M.P.3. Construct viable arguments and critique the reasoning of others. Understand that multiplication and division are related equations, and explain how the dividend in a division equation is the same as the product in a related multiplication equation. For example, $48 \div 6 = \square$ is equivalent to $\square \times 6 = 48$.

M.P.7. Look for and make use of structure. Understand that a division problem in the form $b \div a$ is equivalent to "a times what number is equal to b?" or "How many groups of a are in b?" For example, $36 \div 9$ is equivalent to "9 times what number is equal to 36?" or $9 \times \square = 36$.

Key Academic Terms:

product, quotient, multiplication, division, unknown-factor, equivalent

Operations and Algebraic Thinking
Multiply and divide within 100.
3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Guiding Questions with Connections to Mathematical Practices:

What does it mean to fluently multiply and divide within 100?

M.P.2. Reason abstractly and quantitatively. Use a combination of known facts, patterns and relationships, and other strategies to strategically, efficiently, and accurately multiply and divide within 100. For example, 4×9 can be found by doubling 9 to get 18, then doubling again to get 36 because $2 \times 2 \times 9 = 4 \times 9$. Another strategy is to think of 4×9 as 1 group of 4 less than 10 groups of 4. Ten groups of 4 is 40, so 9 groups of 4 is $40 - 4 = 36$.

How does knowing one multiplication fact translate into knowing all of the multiplication and division facts for a fact family?

M.P.7. Look for and make use of structure. Relate multiplication and division to reason all of the multiplication and division facts that are related. For example, knowing that $3 \times 6 = 18$ also means knowing that $6 \times 3 = 18$, $18 \div 3 = 6$, and $18 \div 6 = 3$.

Key Academic Terms:

fluently, product, dividend, fact, properties of operations

Operations and Algebraic Thinking
Solve problems involving the four operations, and identify and explain patterns in arithmetic.
3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).)

Guiding Questions with Connections to Mathematical Practices:

How can a two-step word problem be modeled with an equation?

M.P.4. Model with mathematics. Identify important quantities from the context of a situation, and create an equation with a letter standing for the unknown quantity, known as a variable. For example, the situation "Ethan has 3 packages of markers with 4 markers in each pack. Jamal has 17 markers. How many more markers does Jamal have than Ethan?" This situation can be modeled using the equation $3 \times 4 + m = 17$, where the variable m is how many more markers Jamal has than Ethan.

How are expressions and equations with multiple operations solved?

M.P.7. Look for and make use of structure. Use the properties of operations, the order of operations, and the context of the situation to solve two-step expressions and equations. For example, the situation "Marcus has 3 apples. He purchases an additional 2 bags of 8 apples each. How many apples does he have in all?" This situation can be modeled using the expression $3 + 2 \times 8$ and solved by first multiplying 2 and 8 to get 16, then adding 3 for a sum of 19 apples.

How can the reasonableness of an answer be checked using mental computation and estimation?

M.P.3. Construct viable arguments and critique the reasoning of others. Detect possible errors by using estimation and the context of a word problem. For example, the situation "Rachel needs 32 cookies to share with her class, and she has already baked 24 cookies. How many more cookies will Rachel need to bake?" will have an answer close to 10 because 32 is close to 30 and 24 is close to 20 and $30 - 20 = 10$.

Key Academic Terms:

unknown quantity, mental computation, estimation, variable, order of operations, two-step problem, equation, expression, rounding, reasonableness

Operations and Algebraic Thinking
Solve problems involving the four operations, and identify and explain patterns in arithmetic.
3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. Example: Observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

Guiding Questions with Connections to Mathematical Practices:

How can an arithmetic pattern be identified?

M.P.3. Construct viable arguments and critique the reasoning of others. Recognize that when consecutive terms always differ by the same amount, an arithmetic pattern is formed. For example, in the pattern 4, 7, 10, 13, . . ., each term in the pattern differs from the previous term by 3. A visual pattern can also be found in the multiplication table, for example, by shading all of the even numbers. The pattern results in all of the numbers in the second, fourth, sixth, eighth, and tenth columns being shaded.

How can an arithmetic pattern be described?

M.P.1. Make sense of problems and persevere in solving them. Understand that an arithmetic pattern can be described by the starting value and an addition or subtraction rule. The pattern can also be connected to skip-counting and multiplication. For example, the pattern 5, 9, 13, 17, . . . can be described as “start with 5, then add 4 each time,” and the pattern 6, 12, 18, 24, . . . is the same pattern that results when starting at 6 and skip-counting by 6. When using an addition or multiplication table, a visual pattern may be described as well.

Key Academic Terms:

arithmetic pattern, starting value, addition table, multiplication table, consecutive, term, decompose

Number and Operations in Base Ten
Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)
3.NBT.10 Use place value understanding to round whole numbers to the nearest 10 or 100.

Guiding Questions with Connections to Mathematical Practices:

How are numbers rounded to the nearest ten or hundred?

M.P.5. Use appropriate tools strategically. Demonstrate on a number line how to round a given number to the nearest 10 or 100. For example, when rounding 56 to the nearest 10, find the location of 56 on the number line and determine the closest 10 is 60.

What makes “5” significant when rounding to the nearest 10?

M.P.7. Look for and make use of structure. Identify that 5 is significant because it represents the halfway point between two 10s on a number line. For example, on a number line from 10 to 20, the interval between 10 and 20 is divided into ten equal-sized sections that are marked with the numbers 11 through 19. The number 15 is the same distance from 10 as it is from 20 on the number line, so it represents the halfway point between 10 and 20. To the left of 15, all the values are closer to 10, and to the right of 15, all of the values are closer to 20. The value of 15 will round up to 20 because half of the values round to 10 and half of the values round to 20.

What makes “50” significant when rounding to the nearest 100?

M.P.7. Look for and make use of structure. Identify that 50 is significant because it represents the halfway point between two 100s on a number line. For example, on a number line from 300 to 400, the interval is divided into ten equal-sized sections that are marked with the numbers 310, 320, 330, 340, 350, 360, 370, 380, and 390. The number 350 is the same distance from 300 as it is from 400 on the number line, so it represents the halfway point between 300 and 400. To the left of 350 all the values are closer to 300, and to the right of 350 all the values are closer to 400. The value of 350 will round up to 400 because half the values round to 300 and half the values round to 400.

Key Academic Terms:

place value, round, nearest 10, nearest 100, multi-digit, halfway point

Number and Operations in Base Ten
Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)
3.NBT.11 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Guiding Questions with Connections to Mathematical Practices:

How can the sum of two numbers be found?

M.P.5. Use appropriate tools strategically. Demonstrate how to add two numbers using a strategy involving paper and pencil, tools such as base-ten blocks, place value, properties of operations, and/or the relationship between addition and subtraction. This prepares students to learn the standard algorithm, which is introduced in grade 4. For example, add 199 + 57 by demonstrating it is the same as 200 + 57 – 1.

How can the difference of two numbers be found?

M.P.5. Use appropriate tools strategically. Demonstrate how to subtract two numbers using a strategy involving paper and pencil, tools such as base-ten blocks, place value, properties of operations, and/or the relationship between addition and subtraction. For example, subtract 341 – 236 by thinking of it as 236 + ___ = 341 and adding up on a number line or by taking 236 away from 341.

Key Academic Terms:

sum, difference, base-ten blocks, place value, multi-digit, properties of operations, algorithm

Number and Operations in Base Ten
Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)
3.NBT.12 Multiply one-digit whole numbers by multiples of 10 in the range 10 – 90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Guiding Question with Connections to Mathematical Practices:

How does multiplying a one-digit number by a multiple of 10 connect to multiplying two one-digit numbers?

M.P.8. Look for and express regularity in repeated reasoning. Extend multiplication of two one-digit numbers to multiplying a one-digit number by a multiple of 10 using understanding of place value and properties of operations. For example, know that because $6 \times 4 = 24$, 6×40 can be thought of as 6×4 tens, which is 24 tens or 240. Or, 4×70 is the same as $4 \times (7 \times 10)$ which equals $(4 \times 7) \times 10$ or $28 \times 10 = 280$.

Key Academic Terms:

multiply, one-digit, multiple of 10, place value, properties of operations

Number and Operations – Fractions
Develop understanding of fractions as numbers.
3.NF.13 Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by *a* parts and size $\frac{1}{b}$.

Guiding Questions with Connections to Mathematical Practices:

What do the numerator and denominator of a fraction represent?

M.P.2. Reason abstractly and quantitatively. Explain that the denominator of a fraction shows how many equal-sized parts a whole is partitioned into, and that the numerator of a fraction shows the number of parts. For example, the fraction $\frac{3}{4}$ refers to 3 parts of a whole that was partitioned into 4 equal parts.

How can the unit fraction be determined from a visual representation?

M.P.2. Reason abstractly and quantitatively. Explain that when a whole is divided into several equal-sized parts, each of those parts represents 1 part of the whole which is written with 1 as the numerator and the number of equal parts as the denominator. For example, if a circle is partitioned into 8 equal pieces, each piece represents $\frac{1}{8}$ of the circle.

How does composing a fraction compare to composing whole numbers?

M.P.7. Look for and make use of structure. Explain that just as any whole number can be made of ones, a fraction is composed of unit fractions of a certain size. For example, $\frac{7}{6}$ is composed of 7 one-sixth pieces, just like 40 is composed of 40 ones.

How can the whole of a fraction be determined?

M.P.6. Attend to precision. Determine the whole in order to determine the unit fraction to use. For example, given 2 circles with each partitioned into 4 equal parts and 7 parts shaded, it should be determined if the whole is 1 circle or 2 circles. If the whole is one circle, the unit fraction is $\frac{1}{4}$, so the fraction represented is $\frac{7}{4}$. If the whole is two circles, the unit fraction is $\frac{1}{8}$, so the fraction represented is $\frac{7}{8}$.

Key Academic Terms:

fraction, numerator, denominator, divide, part, whole, unit fraction, partition, equal-sized parts

Number and Operations – Fractions
Develop understanding of fractions as numbers.
3.NF.14 Understand a fraction as a number on the number line; represent fractions on a number line diagram. **3.NF.14a** Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

Guiding Questions with Connections to Mathematical Practices:

How is the location of a unit fraction on a number line determined?

M.P.5. Use appropriate tools strategically. Demonstrate how to partition and designate the whole to represent a unit fraction given a blank number line. For example, given the fraction $\frac{1}{4}$, label a number line with zero and one, partition the line into four equal intervals by drawing three tick marks, and explain that $\frac{1}{4}$ is located at the first mark when counting by fourths from the zero.

How is the value of a unit fraction on a number line determined?

M.P.2. Reason abstractly and quantitatively. Determine the value of a unit fraction on a number line given the point that represents the unit fraction on the number line and the whole. For example, if the number line with zero and one marked is partitioned into eight equal parts and the point is located at the first tick mark after the zero, the fraction represents $\frac{1}{8}$.

Key Academic Terms:

number line, partition, tick mark, fraction, numerator, denominator, divide, equal parts, whole, unit fraction, equal intervals, value

Number and Operations – Fractions
Develop understanding of fractions as numbers.
3.NF.14 Understand a fraction as a number on the number line; represent fractions on a number line diagram. **3.NF.14b** Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off *a* lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

Guiding Questions with Connections to Mathematical Practices:

How is the location of any fraction on a number line determined?

M.P.5. Use appropriate tools strategically. Demonstrate how to partition and designate the whole to represent a fraction given a blank number line. For example, given the fraction $\frac{2}{4}$, label a number line with zero and one, partition the line into four equal parts by drawing three tick marks, and explain that $\frac{2}{4}$ is located at the second mark when counting by fourths from the zero.

How is the value of any fraction on a number line determined?

M.P.2. Reason abstractly and quantitatively. Determine the value of a fraction on a number line given a point on a number line and the whole. For example, if the number line is divided into eight equal intervals between zero and 1 and another eight equal intervals between 1 and 2, and the point is located at the tenth tick mark after the zero, count ten units from the zero and state that the fraction represents $\frac{10}{8}$.

Key Academic Terms:

number line, partition, tick mark, fraction, numerator, denominator, divide, equal parts, whole, equal intervals, value

Number and Operations – Fractions
Develop understanding of fractions as numbers.
3.NF.15 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. **3.NF.15a** Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line.

Guiding Question with Connections to Mathematical Practices:

How are two or more fractions recognized as being equivalent?

M.P.5. Use appropriate tools strategically. Explain with a variety of representations that two fractions can be the same size but have different names. For example, recognize that pattern blocks representing $\frac{1}{3}$ are exactly the same size as pattern blocks representing $\frac{2}{6}$; similarly, recognize that $\frac{1}{3}$ and $\frac{2}{6}$ are at the same location on a number line.

Key Academic Terms:

equivalent, fraction, number line, point, compare

Number and Operations – Fractions
Develop understanding of fractions as numbers.
3.NF.15 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. **3.NF.15b** Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

Guiding Question with Connections to Mathematical Practices:

How can equivalent fractions be found for a given fraction?

M.P.6. Attend to precision. Explain and demonstrate how equivalent fractions can be found for a given fraction. For example, use a fraction strip folded into two parts with one part shaded to represent $\frac{1}{2}$. Fold the fraction strip in half again so there are four parts with two parts shaded. The fractions $\frac{1}{2}$ and $\frac{2}{4}$ both represent the same quantity of shading, so $\frac{1}{2}$ must equal $\frac{2}{4}$. Fold the fraction strip in half again to demonstrate $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$. Another example would be to use a letter fold to divide the shaded fraction strip so there are six parts with three parts shaded to show that $\frac{1}{2} = \frac{3}{6}$.

Key Academic Terms:

equivalent fractions, model, fraction strip

Number and Operations – Fractions
Develop understanding of fractions as numbers.
3.NF.15 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. **3.NF.15c** Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = $\frac{3}{1}$; recognize that $\frac{6}{1}$ = 6; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.

Guiding Questions with Connections to Mathematical Practices:

How are whole numbers written as fractions?

M.P.2. Reason abstractly and quantitatively. Write any whole number as an equivalent fraction and represent the fraction in a variety of ways. For example, $2 = \frac{4}{2}$, because 2 and $\frac{4}{2}$ are located at the same point on a number line. Also, recognize that $\frac{1}{1}$ is one part of a whole that is divided into 1 part and is equal to 1. Similarly, $\frac{2}{1}$ is "two wholes" and is equal to 2.

What fractions are equivalent to 1?

M.P.7. Look for and make use of structure. Recognize that any fraction that has the same nonzero numerator and denominator is equivalent to 1. For example, $\frac{3}{3}$ and $\frac{6}{6}$ are both equivalent to 1, and to each other, because they each represent 1 whole.

M.P.5. Use appropriate tools strategically. Demonstrate the concept described above by using regions and/or a number line. For example, show that pattern blocks representing $\frac{2}{2}$ are exactly the same size as pattern blocks representing $\frac{6}{6}$; similarly, recognize that $\frac{3}{3}$ and $\frac{4}{4}$ are at the same location on a number line, which is also the location of 1 on a number line.

Key Academic Terms:

equivalent, fractions, whole, numerator, denominator, region, number line, nonzero

Number and Operations – Fractions
Develop understanding of fractions as numbers.
3.NF.15 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. **3.NF.15d** Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Guiding Questions with Connections to Mathematical Practices:

How can fractions with the same denominator be compared?

M.P.3. Construct viable arguments and critique the reasoning of others. Use models to show that the numerator of a fraction indicates the number of parts, so if the denominators of two fractions are the same, the fraction with the greater numerator is the greater fraction. For example, in comparing $\frac{2}{6}$ to $\frac{1}{6}$, demonstrate with fraction models that $\frac{2}{6}$ is the greater fraction because it represents 2 of 6 parts compared to $\frac{1}{6}$ which represents 1 of those same 6 parts.

How can fractions with the same numerator be compared?

M.P.3. Construct viable arguments and critique the reasoning. Use models to show that the denominator of a fraction indicates the size of (equal) parts a whole is partitioned into, and that the greater the denominator, the smaller the parts. For example, in comparing $\frac{2}{3}$ to $\frac{2}{6}$, demonstrate with fraction models that $\frac{2}{3}$ is the greater fraction because both fractions have an equal number of parts, but thirds are larger than sixths.

When is it appropriate/not appropriate to compare two fractions?

M.P.2. Reason abstractly and quantitatively. Determine whether the fractions are describing the same whole before comparing them. For example, $\frac{3}{4}$ of a small sandwich is not the same as $\frac{3}{4}$ of a large sandwich because the whole is not the same.

How can symbols be used to record the comparison of two fractions?

M.P.6. Attend to precision. Record the comparison of two fractions using $>$, $=$, or $<$. For example, $\frac{2}{3} > \frac{2}{6}$.

Key Academic Terms:

compare, fractions, numerator, denominator, part, whole, greater than ($>$), less than ($<$), equal ($=$), fraction models

Measurement and Data
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
3.MD.16 Tell and write time to the nearest minute, and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

Guiding Questions with Connections to Mathematical Practices:

How is an analog clock used to tell time to the nearest minute?

M.P.6. Attend to precision. Explain that one of the wholes used to represent time is the circle of the clock face. The clock face is divided in two ways, by the hour and by the minute. The circular whole is divided into 12 parts where each part is one hour. The number of hours is represented by the short hand. Recognize that the hour is considered a whole which is also represented by the circle of the clock face. Each hour is divided into 60 parts and each part is one minute. The number of minutes is represented by the long hand. For example, if the short hand is between 6 and 7, and the long hand is 17 tick marks past 12, then the time is 6:17.

M.P.6. Attend to precision. Explore relationships within the clock to tell time, paying particular attention to using fractions on the clock face. For example, recognize that when the minute hand is at the 3, it is 15 minutes, or a quarter of the whole, past the hour.

How can a number line be used to solve word problems that involve time intervals?

M.P.5. Use appropriate tools strategically. Skip-count backward or forward on a number line to determine the time length of an event or the time an event begins or ends. For example, if Jose had a 25-minute trumpet lesson that ended at 4:15, then the time the lesson began can be determined by creating an open number line that starts at 4:15, doing a left jump of 15 and marking that as 4:00, then doing another left jump of 10 and marking that as 3:50, paying attention to the fact that 4:00 is equivalent to 3:00 plus 60 minutes.

Key Academic Terms:

analog clock, interval, number line, quarter of the whole, quarter past the hour, tick mark

Measurement and Data
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
3.MD.17 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes multiplicative comparison problems (problems involving notions of "times as much").)

Guiding Questions with Connections to Mathematical Practices:

What is capacity and how is it measured?

M.P.2. Reason abstractly and quantitatively. Understand that capacity indicates the measure of volume (dry or liquid) in a container. For example, the capacity of a jar is 1 liter if it can hold 1 liter of a pourable substance.

What is mass and how is it measured?

M.P.2. Reason abstractly and quantitatively. Understand that mass indicates the amount of matter in an object and can be represented with different-sized units such as grams or kilograms. For example, the mass of smaller objects such as pencils or paper clips is generally recorded in grams while the mass of larger objects such as desks or doors is recorded in kilograms.

How can problems involving masses or volumes with the same unit be solved?

M.P.2. Reason abstractly and quantitatively. Extend previous knowledge of the four operations to add, subtract, multiply, and divide measures of volume and mass. For example, if the mass of one shoe is 335 grams, then the mass of the pair of shoes is 670 grams because $2 \times 335 = 670$.

Key Academic Terms:

volume, capacity, mass, matter, liter, gram, kilogram

Measurement and Data
Represent and interpret data.
3.MD.18 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. Example: Draw a bar graph in which each square in the bar graph might represent 5 pets.

Guiding Questions with Connections to Mathematical Practices:

When drawing a bar graph or picture graph, what is "scale" and why is it important?

M.P.3. Construct viable arguments and critique the reasoning of others. Define scale and use it to create a graph. For example, when given data, create a picture graph where 1 picture of an insect represents 3 actual insects. Explain the importance of noting the scale when comparing or finding the actual values of the data representation.

How are bar graphs and picture graphs connected?

M.P.7. Look for and make use of structure. Notice the similarities and differences between graphed data. For example, note that both bar graphs and picture graphs have a scale, and the scale for bar graphs is labeled on either the horizontal or vertical axis while the scale for picture graphs is given in a pictorial key.

How can the data shown on a graph be used to solve problems involving quantities?

M.P.2. Reason abstractly and quantitatively. Interpret the relationships within data by solving problems to compare quantities. For example, if a bar graph shows that the most popular pet is a dog and the least popular pet is a bird, then the difference between the two bar lengths indicates how many more people prefer dogs than prefer birds.

Key Academic Terms:

bar graph, data, scale, picture graph, horizontal axis, vertical axis, pictorial key

Measurement and Data
Represent and interpret data.
3.MD.19 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters.

Guiding Questions with Connections to Mathematical Practices:

How is a ruler marked in halves and fourths of an inch used to measure length of an object?

M.P.5. Use appropriate tools strategically. Recognize that the tick marks between two consecutive whole numbers on a ruler are equally spaced so that distances of halves and fourths can be measured. For example, the tick mark that is exactly halfway between the numbers 2 and 3 on a ruler indicates a length of $2\frac{1}{2}$ inches.

What is a line plot and how can it be used to show data involving whole numbers, halves, and quarters?

M.P.4. Model with mathematics. Understand that a line plot is a graph that displays a distribution of data values, including whole numbers, halves, and quarters, such that each data value is marked above a horizontal line with an X or dot. For example, if a box of paper clips includes 3 clips that are each $\frac{2}{4}$ of an inch long, 4 clips that are each $\frac{3}{4}$ of an inch long, and 2 clips that are each 1-inch long, then a line plot can be constructed with 3 Xs stacked vertically above $\frac{2}{4}$, 4 Xs stacked vertically above $\frac{3}{4}$, and 2 Xs stacked vertically above 1.

Key Academic Terms:

ruler, data, line plot, horizontal scale, distribution of data, halves, fourths, quarters

Measurement and Data
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
3.MD.20 Recognize area as an attribute of plane figures, and understand concepts of area measurement. **3.MD.20a** A square with side length 1 unit called “a unit square,” is said to have “one square unit” of area and can be used to measure area.

Guiding Questions with Connections to Mathematical Practices:

What is area?

M.P.7. Look for and make use of structure. Recognize that area is a measure of the size of a surface. For example, the area of a piece of paper can be measured by counting or calculating the number of identical squares required to cover one side of the paper.

What is a unit square and how can it be used to measure area?

M.P.2. Reason abstractly and quantitatively. Understand that a unit square is a square with a side length of 1 unit, and that such a square represents a unit of measurement. For example, if the surface of a plane figure is exactly the same size as the surface area of a square with side length 1 unit, then the shape has an area of 1 square unit.

Key Academic Terms:

area, unit square, surface, attribute, plane figure, square unit

Measurement and Data
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
3.MD.20 Recognize area as an attribute of plane figures, and understand concepts of area measurement. **3.MD.20b** A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

Guiding Question with Connections to Mathematical Practices:

How can multiple unit squares be used to measure area?

M.P.5. Use appropriate tools strategically. Recognize that area is the number of unit squares needed to cover a surface, and understand that multiple unit squares can be combined to measure the area of plane figures so long as the unit squares completely cover the figure without overlapping each other or extending beyond the edge of the figure. For example, if the surface of a flat rectangular ruler can be covered with 12 squares of side length 1 unit, then the ruler has an area of 12 square units.

Key Academic Terms:

area, unit square, plane figure, attribute, square unit, gap, overlap, edge

Measurement and Data
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
3.MD.21 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

Guiding Question with Connections to Mathematical Practices:

How can graph paper with square grids be used to measure the area of plane figures?

M.P.5. Use appropriate tools strategically. Determine the area of plane figures drawn on graph paper by counting the number of squares within the figure and draw figures of a given area. For example, if a rectangle made of 2 rows of 4 squares is drawn on graph paper, then the area of the rectangle is 8 square units because there is a total of 8 complete squares within the rectangle.

Key Academic Terms:

area, unit square, plane figure, square centimeters, square meters, square inch, square feet

Measurement and Data
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
3.MD.22 Relate area to the operations of multiplication and addition. **3.MD.22a** Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

Guiding Questions with Connections to Mathematical Practices:

What is tiling and how is it used to determine the area of a rectangle with whole-number side lengths?

M.P.4. Model with mathematics. Understand tiling as the covering of an entire plane figure with nonoverlapping regular shapes, and that if the shapes are unit squares, then the total number of squares covering the plane figure will represent the area of the figure. For example, if a rectangle with side lengths of 3 inches and 6 inches is "tiled" into unit squares by dividing it into 3 equal-sized columns and 6 equal-sized rows, then the area of the rectangle is 18 square inches because there is a total of 18 one-inch unit squares.

How is tiling a rectangle that has whole-number side lengths related to arrays and multiplication?

M.P.2. Reason abstractly and quantitatively. Recognize that tiling a rectangle that has whole-number side lengths with unit squares produces rows and columns, and that multiplying the number of rows by the number of columns is equivalent to the total number of squares just like arrays. For example, if a rectangle with side lengths of 5 inches and 2 inches is divided into 5 columns and 2 rows, then there is an array of 2 rows of 5 squares, which is the same as multiplying 2×5 or counting the squares one at a time.

Key Academic Terms:

area, rectangle, tiling, array, nonoverlapping, row, column, equivalent

Measurement and Data
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
3.MD.22 Relate area to the operations of multiplication and addition. **3.MD.22b** Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

Guiding Question with Connections to Mathematical Practices:

How can multiplication be used to determine the area of rectangles?

M.P.2. Reason abstractly and quantitatively. Extend previous understanding of multiplication to determine the area of rectangular figures by finding the product of the two side lengths. For example, if the side lengths of a rectangular doormat are 2 feet and 3 feet, then finding the product of 2 and 3 can be used to determine that the doormat has an area of 6 square feet.

Key Academic Terms:

area, multiplication, product, rectangle, whole number

Measurement and Data
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
3.MD.22 Relate area to the operations of multiplication and addition. **3.MD.22c** Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning.

Guiding Question with Connections to Mathematical Practices:

How can a tiled rectangle with whole-number side lengths be decomposed?

M.P.7. Look for and make use of structure. Recognize that the side length of a rectangle can be rewritten as the sum of two numbers, and that when the other side is multiplied by each of those two numbers, then the sum of the products is equal to the area of the rectangle. For example, if a rectangle has side lengths of 4 units and 8 units, then 4 can be rewritten as the sum of 2 and 2. As a result, a tiled rectangle with 2 rows and 8 columns is created along with another tiled rectangle with 2 rows and 8 columns. The total number of tiles is equal to the sum of (8×2) and (8×2), which is the same as $16 + 16 = 32$.

Key Academic Terms:

area, rectangle, tiling, multiplication, addition, decompose

Measurement and Data
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
3.MD.22 Relate area to the operations of multiplication and addition. **3.MD.22d** Recognize area as additive. Find areas of rectilinear figures by decomposing them into nonoverlapping rectangles and adding the areas of the nonoverlapping parts, applying this technique to solve real-world problems.

Guiding Question with Connections to Mathematical Practices:

How can polygons with all right angles be decomposed to find area?

M.P.7. Look for and make use of structure. Recognize that rectilinear shapes can be decomposed into nonoverlapping rectangles, and that the sum of the areas of the nonoverlapping rectangles is equivalent to the area of the original rectilinear shape. For example, a *U*-shaped figure that is 5 units tall and 4 units wide, with an opening that is 4 units tall and 2 units wide, can be decomposed into 3 rectangles that are each 4 units by 1 unit.

Key Academic Terms:

decompose, area, additive, rectilinear, equivalent, nonoverlapping

Measurement and Data
Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
3.MD.23 Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Guiding Questions with Connections to Mathematical Practices:

What is perimeter and how is it determined?

M.P.2. Reason abstractly and quantitatively. Understand perimeter as the distance around a shape, and recognize that perimeter can be calculated by adding all of the sides of the shape together. For example, a yard that is shaped like a triangle with side lengths of 30 feet, 40 feet, and 50 feet has a perimeter of 120 feet because $30 + 40 + 50 = 120$, so 120 feet of fencing would be needed to surround the yard.

When can multiplication be used to determine the perimeter of a figure?

M.P.2. Reason abstractly and quantitatively. Recognize that if all the sides of a polygon are equal, then the perimeter can be determined by multiplying one side length by the total number of sides. For example, a square with side lengths of 4 inches has a perimeter of 16 inches because $4 \times 4 = 16$, which is equivalent to $4 + 4 + 4 + 4$.

Do shapes with the same area have the same perimeter and vice versa?

M.P.2. Reason abstractly and quantitatively. Recognize that common perimeters do not indicate common areas nor do common areas indicate common perimeters. For example, although a rectangle with side lengths of 3 units and 4 units has the same area as a rectangle with side lengths of 2 units and 6 units, the perimeter of the former is 14 units while the perimeter of the latter is 16 units. In the same way, although a rectangle with side lengths of 3 units and 5 units has the same perimeter as a rectangle with side lengths of 2 units and 6 units, the area of the former is 15 square units while the area of the latter is 12 square units.

Key Academic Terms:

perimeter, polygon, area, equivalent

Geometry
Reason with shapes and their attributes.
3.G.24 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Guiding Questions with Connections to Mathematical Practices:

How can the attributes of a shape help to categorize the shape?

M.P.7. Look for and make use of structure. Explore the attributes of a shape to make decisions about how to categorize the shape. For example, any shape with three straight sides is a triangle, and any shape with four straight sides is a quadrilateral.

How can a shape be a quadrilateral and not belong to another subcategory of quadrilaterals (rhombus, rectangle, or square)?

M.P.3. Construct viable arguments and critique the reasoning of others. Identify the attributes that are needed to belong to the subcategories of rhombuses, rectangles, and squares, and recognize when a shape does not have those attributes. For example, a quadrilateral with all four sides of different lengths will not be a rhombus, rectangle, or square.

Why is a square always categorized as a rectangle, but a rectangle is not always categorized as a square?

M.P.6. Attend to precision. Observe that a square has four sides of equal length and four angles of equal measure, but a rectangle only needs the pair of opposite sides to be of equal length and all four angles to be equal. As such, the definition of a rectangle is less restrictive than the definition of a square. For example, a quadrilateral with four equal angles and sides of 5 inches in length can be called both a square and a rectangle, but the square categorization is more specific than the rectangle categorization.

Key Academic Terms:

shape, attributes, category, sub-category, rhombus, rectangle, square, parallelogram, trapezoid, quadrilateral, angle, opposite sides

Geometry
Reason with shapes and their attributes.
3.G.25 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. Example: Partition a shape into 4 parts with equal area, and describe the area of each part as $\frac{1}{4}$ of the area of the shape.

Guiding Questions with Connections to Mathematical Practices:

How can a shape be decomposed into parts with equal areas?

M.P.2. Reason abstractly and quantitatively. Use spatial reasoning to find ways to decompose shapes into equal partitions of same and different shapes. For example, equal-sized slices of a circle can be accomplished by drawing lines that all pass through the center of the circle.

How can fractions describe the partitions of a shape?

M.P.7. Look for and make use of structure. Connect a partitioned shape to fractions of a whole, with the denominator being the total number of partitions of equal area. For example, when a rectangle is partitioned into 8 equal-sized but differently-shaped quadrilaterals, each will represent $\frac{1}{8}$ of the area of the whole rectangle.

Key Academic Terms:

shape, area, partition, fraction, decompose