ACAP
Alabama Comprehensive Assessment Program

SUMMATIVE

Grade 4
Mathematics

Alabama Educator Instructional Supports

Alabama Course of Study Standards

DATA RECOGNITION CORPORATION

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Introduction

The *Alabama Course of Study Instructional Supports: Math* is a companion manual to the 2016 *Revised Alabama Course of Study: Math* for Grades K–12. Instructional supports are foundational tools teachers may use to help students become independent learners as they build toward mastery of the *Alabama Course of Study* content standards.

- The purpose of the instructional supports found in this manual is to help teachers engage their students in exploring, explaining, and expanding their understanding of the content standards.

The content standards contained within the course of study may be accessed on the Alabama State Department of Education (ALSDE) website at www.alsde.edu.

Educators are reminded that content standards indicate minimum content—what all students should know and be able to do by the end of each grade level or course. Local school systems may have additional instructional or achievement expectations and may provide instructional guidelines that address content sequence, review, and remediation.

Organization

The organizational components of this manual include standards, guiding questions, connections to instructional supports, key academic terms, and examples of activities. The definition of each component is provided below:

Content Standard:	The content standard is the statement that defines what all students should know and be able to do at the conclusion of a given grade level or course. Content Standards contain minimum required content and complete the phrase "Students will."
Guiding Questions:	Each guiding question is designed to create a framework for the given standard. Therefore, each question is written to help teachers convey important concepts within the standard. By utilizing guiding questions, teachers are engaging students in investigating, analyzing, and demonstrating knowledge of the underlying concepts reflected in the standard.

Connection to Instructional Supports: The purpose of each instructional support is to engage students in exploring, explaining, and expanding their understanding of the content standards provided in the 2016 *Revised Alabama Course of Study: Math*. An emphasis is placed on the integration of the eight Standards for Mathematical Practice.

Mathematical Practices

The Standards for Mathematical Practice describe the varieties of expertise that mathematics educators at all levels should seek to develop in their students. They rest on the National Council of Teachers of Mathematics (NCTM) process standards and the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up: Helping Children Learn Mathematics.*

The Standards for Mathematical Practice are the same for all grade levels and are listed below.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Key Academic Terms: The academic terms included in each instructional support. These academic terms are derived from the standards and are to be incorporated into instruction by the teacher and used by the students.

Instructional Activities: A representative set of sample activities and examples that can be used in the classroom. The set of activities and examples is not intended to include all the activities and examples defined by the standard. These will be available in Fall 2020.

Additional Resources: Additional resources include resources that are aligned to the standard and may provide additional instructional strategies to help students build toward mastery of the designated standard. These will be available in Fall 2020.

Operations and Algebraic Thinking
Use the four operations with whole numbers to solve problems.
4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

Guiding Questions with Connections to Mathematical Practices:

How are multiplicative comparisons similar to and different from equal groups and arrays?

M.P.1. Make sense of problems and persevere in solving them. Describe with drawings and words the ways multiplicative comparisons are similar to and different from equal groups and arrays. For example, both problem types use multiplication to solve problems, but equal groups and arrays are focused on repeated addition (3 groups of 4 is 12), and multiplicative comparisons are interpreted as a quantity relative to other quantities as a multiple or factor (12 is 3 times as many as 4 or 4 times as many as 3).

How can a multiplicative comparison be written as an equation and a statement?

M.P.2. Reason abstractly and quantitatively. Write equations and verbal statements to represent multiplicative comparisons. For example, read or write $6 \times 4 = 24$ as “24 is 6 times as many as 4,” and write an equation for “14 is twice as much as 7” as $14 = 2 \times 7$.

Key Academic Terms:

multiplicative comparison, times as many, product, factor, verbal statement, repeated addition, multiplication equation, multiple

Operations and Algebraic Thinking
Use the four operations with whole numbers to solve problems.
4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

Guiding Questions with Connections to Mathematical Practices:

How can word problems involving multiplicative comparison be represented and solved?

M.P.4. Model with mathematics. Represent and solve a word problem with drawings and/or equations with an unknown. For example, represent the situation "There are 12 children at the playground and 3 adults. How many times as many children were at the playground than adults?" with the equation $12 = a \times 3$ and a tape diagram with a total of 12 and several groups of 3, repeating each group 4 times to solve.

How is a multiplicative comparison different from an additive comparison?

M.P.3. Construct viable arguments and critique the reasoning of others. Contrast multiplicative and additive comparisons. For example, distinguish that additive comparisons ask what number needs to be added to one quantity to result in the other and multiplicative comparisons ask what factor would multiply one quantity to result in the other.

Key Academic Terms:

multiplicative comparison, times as many, product, factor, multiplication, equation, symbol, additive comparison, tape diagram

Operations and Algebraic Thinking
Use the four operations with whole numbers to solve problems.
4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Guiding Questions with Connections to Mathematical Practices:

How can the appropriate operation(s) be determined for a word problem?

M.P.1. Make sense of problems and persevere in solving them. Plan out and solve multistep word problems using mathematical operations by making sense of the problem. For example, know that a question about making the same amount of money over a span of days and then spending some of the money will involve multiplication and then subtraction.

How does context determine what to do with a remainder?

M.P.2. Reason abstractly and quantitatively. Know that a remainder can be interpreted in many ways, depending on the question being asked. It could be appropriate to ignore the remainder, to round up, or to partition it. For example, if children are sharing marbles equally, remaining marbles should be ignored, since they cannot be partitioned or rounded up.

How can an equation with a letter representing the unknown be used to represent a word problem?

M.P.4. Model with mathematics. Write an equation with a letter representing the missing quantity and solve for that quantity. For example, write the equation $(3 \times 4) + (6 \times 2) = a$ and solve for a in response to the following word problem: "Juan gives 4 apple slices to each of his 3 friends. Then he gives 6 apple slices each to his sister and brother. How many apple slices did Juan give?"

How can the reasonableness of an answer be justified?

M.P.3. Construct viable arguments and critique the reasoning of others. Identify whether an answer is reasonable without solving a problem. For example, for the equation $655 \times 9 = 4{,}585$, round 655 to 700 and solve $700 \times 9 = 6{,}300$ to realize the product is too small.

Key Academic Terms:

addition, subtraction, multiplication, division, operation, multistep problem, remainder, unknown quantity, equation, rounding, mental strategy, partition

Operations and Algebraic Thinking
Gain familiarity with factors and multiples.
4.OA.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

Guiding Questions with Connections to Mathematical Practices:

What patterns can be found in the factor pairs of a number or numbers?

M.P.7. Look for and make use of structure. Describe patterns found in the factor pairs of a single number or multiple numbers. For example, because 18 is a factor of 36, if you know the factors of 18, you can find the factors of 36 by noticing that 36 is double 18 and that, to find the factors of 36, you need to double one factor in the factor pairs of 18. The factor pairs of 18 are 1 × 18, 2 × 9, and 3 × 6. The factor pairs of 36 are 1 × 36, 2 × 18, 3 × 12, 4 × 9, and 6 × 6.

How can all factor pairs be found for a number?

M.P.8. Look for and express regularity in repeated reasoning. Identify a strategy to know when all factor pairs of a number have been found. For example, to find the factor pairs of 12, start with the factor pair including 1 and go through each whole number until the factor pairs reverse, which occurs between 3 × 4 and 4 × 3. Then all factor pairs of 12 (i.e., 1 × 12, 2 × 6, and 3 × 4) have been found.

How can the terms "multiple," "factor," "prime," and "composite" be used to communicate precisely and solve problems?

M.P.6. Attend to precision. Define and use "multiple," "factor," "prime," and "composite" to describe numbers. For example, the factors of 6 are 1, 2, 3, and 6, or 6 is a multiple of 1, 2, 3, and 6. Because 6 has more than two factors, 6 is a composite number. Similarly, the factors of 13 are 1 and 13, or 13 is a multiple of 1 and 13. Because 13 has just two factors, 13 is a prime number.

Key Academic Terms:

multiple, factor, prime, composite, whole number, factor pair

Operations and Algebraic Thinking
Generate and analyze patterns.
4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. Example: Given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence, and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

Guiding Questions with Connections to Mathematical Practices:

How can a pattern be created from a rule?

M.P.7. Look for and make use of structure. Generate a number or shape pattern when given a rule. For example, use the rule "Multiply by 2" and the starting number 1 to generate the sequence 1, 2, 4, 8, 16 . . . and recognize that the rule continues.

How can the properties of a rule or pattern be used to extend a pattern?

M.P.8. Look for and express regularity in repeated reasoning. Analyze a rule or pattern of numbers or shapes to extend the pattern. For example, given a sequence of square designs where each design has two more squares than the previous and the first design has 1 square, reason about how the squares are organized to determine that the 10th design has 19 squares.

What are some features of a given pattern that are not explicit in the pattern's rule?

M.P.3. Construct viable arguments and critique the reasoning of others. Analyze a pattern to identify features of the pattern that are not explicit in the rule. For example, using the pattern 2, 6, 18, 54 . . . from the rule "Multiply by 3" and the starting number 2, identify that all the numbers are even and are also multiples of 3, except for 2.

Key Academic Terms:

generate, rule, pattern, sequence, term, continue, identify, explicit

Numbers and Operations in Base Ten
Generalize place value understanding for multi-digit whole numbers.
4.NBT.6 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. Example: Recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.

Guiding Question with Connections to Mathematical Practices:

What happens to the values the digits represent when moving to the right and left in a multi-digit number?

M.P.2. Reason abstractly and quantitatively. Understand that each place value represents a different-sized unit and that, when comparing the place values of two digits that are next to each other, the place value of the digit on the right is $\frac{1}{10}$ the place value of the digit on the left. The place value of the digit on the left is 10 times the place value of the digit on the right. For example, when multiplying 813 by 10, the value of each digit is multiplied by 10, into one greater place value. The 8 in the hundreds place becomes 8 thousands, 1 ten becomes 1 hundred, 3 ones become 3 tens, and there are 0 ones.

Key Academic Terms:

place value, division, multiplication, multi-digit

Numbers and Operations in Base Ten
Generalize place value understanding for multi-digit whole numbers.
4.NBT.7 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Guiding Questions with Connections to Mathematical Practices:

How are commas used to indicate place values for multi-digit whole numbers?

M.P.7. Look for and make use of structure. Explain the structure of the base-ten system to help read and write whole numbers in a variety of ways. For example, when reading 328,908, notice that the first group of three numbers is read as hundreds, tens, and then ones, followed by thousands (the base-thousand unit). The second group of three numbers is also read as hundreds, tens, and ones, giving the entire phrase the following value: three hundred twenty-eight thousand nine hundred eight.

How are number names related to the expanded form of a number?

M.P.7. Look for and make use of structure. Explain how knowing the number name can be used to write the expanded form of the number by identifying what power of 10 should be multiplied by each digit to generate the value of the digit in expanded form. For example, 275,824 can be divided into 275 in the thousands period and 824 in the ones period. The 275 in the thousands period can be expanded to $(200 + 70 + 5) \times 1{,}000$, and 824 can be expanded to $800 + 20 + 4$. By using properties of operations, the thousands period can be simplified to become the expanded form of $200 \times 1{,}000 + 70 \times 1{,}000 + 5 \times 1{,}000$, or $200{,}000 + 70{,}000 + 5{,}000$. Combining this with the expansion of 824 results in $200{,}000 + 70{,}000 + 5{,}000 + 800 + 20 + 4$.

How can multi-digit numbers be compared?

M.P.8. Look for and express regularity in repeated reasoning. Explain the connection between comparisons of two numbers of any value to comparisons made in previous learning. For example, when comparing 1,428 and 2,093, recognize that, starting with the highest place value, $1{,}428 < 2{,}093$ because 1,000 is less than 2,000.

Key Academic Terms:

base-ten numerals, expanded form, number name, comma, place value, properties of operations, period

Numbers and Operations in Base Ten
Generalize place value understanding for multi-digit whole numbers.
4.NBT.8 Use place value understanding to round multi-digit whole numbers to any place.

Guiding Questions with Connections to Mathematical Practices:

How is rounding past the hundreds place related to rounding to the tens and hundreds?

M.P.8. Look for and express regularity in repeated reasoning. Extend rounding to more place values by using similar methods to previous learning about rounding. For example, use a number line strategy to round to tens and hundreds and then use the same strategy to round to thousands.

What makes the digit "5" in the hundreds place significant when rounding to the nearest thousand?

M.P.7. Look for and make use of structure. Identify that the digit 5 in the hundreds place is significant because it represents the halfway point between two thousands on a number line. For example, on a number line from 3,000 to 4,000, the interval between 3,000 and 4,000 is divided into ten equal-sized sections that are marked with the numbers 3,100 to 3,900 counting by one hundreds. The number 3,500 is the same distance from 3,000 as it is from 4,000 on the number line, so it represents the halfway point between 3,000 and 4,000. To the left of 3,500, all the values are closer to 3,000, and to the right of 3,500, all of the values are closer to 4,000. The value of 3,500 will round up to 4,000 because half of the values round to 3,000 and half of the values round to 4,000. The value 3,500 is the halfway point, so the digit 5 in the hundreds place is significant when rounding to the thousands. The same is true when rounding to any place value; the digit 5 is significant when rounding to a place value that is 10 times the value of the placement of the 5.

Key Academic Terms:

round, place value, ones, tens, hundreds, thousands, ten thousands, hundred thousands, millions, about, approximately, halfway point

Numbers and Operations in Base Ten
Use place value understanding and properties of operations to perform multi-digit arithmetic.
4.NBT.9 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Guiding Questions with Connections to Mathematical Practices:

Why does the standard algorithm for addition and subtraction work?

M.P.3. Construct viable arguments and critique the reasoning of others. Use the standard algorithms for addition and subtraction to solve problem involving multi-digit whole numbers with regrouping, and explain why the standard algorithm works. For example, while subtracting with the standard algorithm, it is important to remember that the following concept can be applied when regrouping: one ten is equivalent to a bundle of ten ones, and one hundred is equivalent to a bundle of ten tens, and so on.

How does the standard algorithm connect with other addition and subtraction strategies?

M.P.1. Make sense of problems and persevere in solving them. Connect the algorithm and other addition strategies to make sense of the algorithm. For example, when solving 1,326 + 46, students should understand that placing the 1 above the tens place means a ten is being added.

Where are numbers decomposed and composed in the standard algorithm?

M.P.1. Make sense of problems and persevere in solving them. Identify when and where numbers are decomposed in the standard algorithm to gain insight into the algorithm. For example, when adding 398 + 84, adding 4 + 8 makes 12. The 12 is decomposed into 10 and 2, and the 2 ones stay in the ones place and the 1 ten goes to the tens place. Then, 1 ten + 9 tens + 8 tens equals 18 tens. The 18 tens are decomposed as a 10 and an 8. The 8 tens stay in the tens place and the 10 tens goes to the hundreds place for 1 hundred. Finally, 1 hundred + 3 hundreds equals 4 hundreds.

Key Academic Terms:

addition, subtraction, standard algorithm, place value, decompose, compose

Numbers and Operations in Base Ten
Use place value understanding and properties of operations to perform multi-digit arithmetic.
4.NBT.10 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Guiding Questions with Connections to Mathematical Practices:

How are area models and arrays related to equations and multiplication strategies?

M.P.4. Model with mathematics. Connect area models and arrays to equations and multiplication strategies to explain and illustrate a calculation. For example, when solving 43×22, draw an area model of $(40 + 3) \times (20 + 2)$. Connect this area model to the partial product expression $(40 \times 20) + (3 \times 20) + (40 \times 2) + (3 \times 2)$.

How do knowledge of place value and properties of operations help solve multiplication problems?

M.P.2. Reason abstractly and quantitatively. Decompose and compose numbers in a variety of ways using place value and the properties of operations to demonstrate a variety of different strategies that use known facts. For example, students can see that multiplication of a multi-digit number by a one-digit number can be simplified into a one-digit number by one-digit number multiplication problem multiplied by a multiple of 10, such as the multiplication problem $364 \times 8 = (3 \times 8 \times 100) + (6 \times 8 \times 10) + (4 \times 8)$.

What strategy makes the most sense to use to solve a given multiplication problem and why?

M.P.3. Construct viable arguments and critique the reasoning of others. Analyze a given problem and choose the strategy that provides an entry point. Depending on experiences and understanding, one problem could have many different strategies that would all be appropriate. For example, when solving 35×14, one student may decompose 35 into $30 + 5$ and 14 into $10 + 4$ and solve the problem using four different products. Another student may solve the same problem by doubling/halving to adjust the problem from 35×14 to 70×7.

How is multiplying a two-, three-, or four-digit number by a one-digit number similar to and different from multiplying a two-digit number by a two-digit number?

M.P.8. Look for and express regularity in repeated reasoning. Connect multiplying by a one-digit number to multiplying by a two-digit number to eventually lead to a general method using knowledge of place value and the properties of operations instead of using the standard algorithm for multiplication, which is introduced in grade 5. For example, students compare multiplying 42×8 to multiplying 42×18 and note similarities and differences in the process.

Key Academic Terms:

strategy, multiply, place value, equation, array, area model, partial product, multiple of 10

Numbers and Operations in Base Ten
Use place value understanding and properties of operations to perform multi-digit arithmetic.
4.NBT.11 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Guiding Questions with Connections to Mathematical Practices:

How can multiplication be used to solve division problems?

M.P.2. Reason abstractly and quantitatively. Describe division as an unknown factor problem, and use the greatest multiple strategy to solve. For example, when solving $122 \div 3$, rewrite as $3 \times n = 122$. The greatest multiple of 3 that goes into 12 tens and 2 ones is 4 tens with 2 ones remaining, so the answer is 40 with a remainder of 2.

How are area models and arrays related to equations and division strategies?

M.P.4. Model with mathematics. Connect area models and arrays to equations and division strategies to explain and illustrate a calculation. For example, when solving $762 \div 6$, use an area model to show that in a rectangle with the total area of 762 and a side length of 6, the length of the unknown side is 1 hundred, 2 tens, and 7 ones. Show this by decomposing 762 to $600 + 120 + 42$. Then write equations that correspond with the model (e.g., $762 - (6 \times 100) = 162$; $162 - (6 \times 20) = 42$; $42 - (6 \times 7) = 0$).

How do knowledge of place value and properties of operations help solve division problems?

M.P.2. Reason abstractly and quantitatively. Decompose and compose numbers in a variety of ways using place value and the properties of operations to demonstrate a variety of different strategies for division. For example, use place value knowledge when solving $3{,}600 \div 9$ to show that 3,600 is the same as 36 hundreds and that 36 hundreds $\div$ 9 is 4 hundreds, or 400.

How does dividing a two-, three-, or four-digit number by a one-digit number connect to dividing numbers within 100?

M.P.8. Look for and express regularity in repeated reasoning. Connect knowledge of dividing numbers within 100 to dividing larger numbers using knowledge of place value, the properties of operations, and the relationship between multiplication and division. This prepares students to learn the standard algorithm for division, which is introduced in grade 6. For example, compute $87 \div 4$, $320 \div 3$, and $2{,}444 \div 6$, noting similarities and differences in the division.

What is the meaning of the remainder of a division problem in context?

M.P.1. Make sense of problems and persevere in solving them. Explain the meaning of the remainder both in and out of context. For example, Evan has 26 stickers and will be giving the same amount to each of his 6 friends. How many stickers will Evan have left? The key to answering this question is to recognize that the remainder is the value needed. Solving $26 \div 6$ is 4 R2. This means Evan will have 2 stickers left.

Key Academic Terms:

quotient, dividend, divisor, divide, multiply, multiple, equation, remainder, area model, greatest multiple, decompose, compose, array, properties of operations, unknown factor

Number and Operations – Fractions
Extend understanding of fraction equivalence and ordering.
4.NF.12 Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{n \times a}{n \times b}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Guiding Questions with Connections to Mathematical Practices:

How can fractions with different denominators and numerators be equivalent?

M.P.3. Construct viable arguments and critique the reasoning of others. Demonstrate that two fractions are equivalent, using various fraction models, and demonstrate that even though the number and size of the parts differ, the fractions are the same size. For example, use an area model to explain that $\frac{6}{8} = \frac{3}{4}$ because both fractions shade the same amount of area on the model.

How can equivalence be determined for two fractions?

M.P.8. Look for and express regularity in repeated reasoning. Using visual fraction models, determine that, when a fraction's numerator and denominator are multiplied or divided by $\frac{n}{n}$ to create a new fraction, the new fraction is equivalent to the original. For example, use a tape diagram to find that $\frac{9}{12}$ is equivalent to both $\frac{3}{4}$ and $\frac{6}{8}$ and write the multiplication or division equations that represent the tape diagram: $\frac{9}{12} \div \frac{3}{3} = \frac{3}{4}$ and, therefore, $\frac{9}{12} = \frac{3}{4}$; $\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$ and therefore, $\frac{3}{4} = \frac{6}{8}$.

Key Academic Terms:

fraction, numerator, denominator, equivalent, fraction models, multiples, multiply, area model, tape diagram

Number and Operations – Fractions
Extend understanding of fraction equivalence and ordering.
4.NF.13 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Guiding Questions with Connections to Mathematical Practices:

How can fractions with unlike numerators and denominators be compared?

M.P.1. Make sense of problems and persevere in solving them. Choose a strategy that makes the most sense, such as finding a common numerator or denominator, using a benchmark fraction, or using residual thinking, to compare two given fractions. For example, when comparing $\frac{13}{12}$ and $\frac{6}{5}$, students should notice that both are one fractional part larger than one whole ($\frac{12}{12}+\frac{1}{12}$ and $\frac{5}{5}+\frac{1}{5}$) and $\frac{1}{5}$ is larger than $\frac{1}{12}$, so $\frac{6}{5}$ is greater than $\frac{13}{12}$.

When is it appropriate/not appropriate to compare two fractions?

M.P.2. Reason abstractly and quantitatively. Determine whether the fractions are describing the same whole before comparing them. For example, $\frac{3}{4}$ of a small pizza is not the same as $\frac{9}{12}$ of a large pizza even though $\frac{3}{4}=\frac{9}{12}$ because the whole isn't the same.

How can symbols be used to record the comparison of two fractions with unlike numerators and denominators?

M.P.6. Attend to precision. Record the comparison of two fractions using $<$, $>$, or $=$. For example, $\frac{5}{6}<\frac{11}{12}$.

In what ways can the comparison of two fractions be justified?

M.P.3. Construct viable arguments and critique the reasoning of others. Justify comparisons using objects, drawings, diagrams, equations, and/or words. For example, when comparing $\frac{2}{6}$ and $\frac{1}{5}$, a student may use fraction circles, tape diagrams, or words to find $\frac{2}{6} = \frac{1}{3}$, then use the same-numerator strategy to conclude $\frac{1}{3} > \frac{1}{5}$; therefore, $\frac{2}{6} > \frac{1}{5}$.

Key Academic Terms:

compare, equivalent fractions, numerator, denominator, benchmark fraction, less than (<), greater than (>), equal to (=), common numerator, common denominator, tape diagram, equation, residual thinking, fraction circle

Number and Operations – Fractions
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.14 Understand a fraction $\frac{a}{b}$ with *a* > 1 as a sum of fractions $\frac{1}{b}$. **4.NF.14a** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

Guiding Questions with Connections to Mathematical Practices:

How many of $\frac{1}{b}$ are added to compose $\frac{a}{b}$?

M.P.2. Reason abstractly and quantitatively. Explain that any fraction, $\frac{a}{b}$, is the sum of *a* of the unit fraction $\frac{1}{b}$. For example, $\frac{3}{2}$ is the sum of 3 of the unit fraction $\frac{1}{2}$.

How is adding and subtracting fractions with the same denominator connected to adding and subtracting whole numbers?

M.P.7. Look for and make use of structure. Explain that addition is joining parts and subtraction is separating parts for both whole numbers and fractions. For example, representing $2 + 3$ on a number line as joining 2 units and 3 units is similar to representing $\frac{2}{8} + \frac{3}{8}$ as joining $\frac{2}{8}$ of a unit and $\frac{3}{8}$ of the same unit.

When is it appropriate/not appropriate to add and subtract fractions?

M.P.3. Construct viable arguments and critique the reasoning of others. Create examples and counterexamples of situations in which adding and subtracting fractions is appropriate. For example, when a student is given fractions without context, the unit is assumed to be the same whole for all of them, and those fractions can always be added and subtracted no matter the denominator. A counterexample is the following situation: “Jayquan has $\frac{3}{4}$ of a cup of milk. He drinks $\frac{1}{4}$ of his milk. How much milk remains?” This problem cannot be solved by subtracting the fractions because $\frac{1}{4}$ of $\frac{3}{4}$ cups of milk is not equal to $\frac{1}{4}$ cup.

Key Academic Terms:

fractions, addition, subtraction, sum, whole numbers, numerator, denominator, whole, equations, unit fraction, compose

Number and Operations – Fractions
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.14 Understand a fraction $\frac{a}{b}$ with *a* > 1 as a sum of fractions $\frac{1}{b}$. **4.NF.14b** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$.

Guiding Questions with Connections to Mathematical Practices:

How is decomposing a fraction connected to decomposing a whole number?

M.P.7. Look for and make use of structure. Explain the connection between the decomposition of fractions and whole numbers. For example, both whole numbers and fractions can be decomposed into addition equations in a variety of ways.

How can addition and subtraction equations be used to convert between mixed numbers and fractions greater than 1?

M.P.2. Reason abstractly and quantitatively. Explain and show addition equations to find equivalence between mixed numbers and fractions greater than 1. For example, $\frac{8}{3} = \frac{3}{3} + \frac{3}{3} + \frac{2}{3} = 1 + 1 + \frac{2}{3} = 2\frac{2}{3}$ and vice versa.

How can drawings or visual models be used to justify the decomposition of a fraction?

M.P.4. Model with mathematics. Identify relationships between models and equations to justify decompositions. For example, use fraction manipulatives to show how many $\frac{1}{6}$ parts are in $\frac{5}{6}$, and then write an equation to represent the manipulatives.

How can any fraction, whole number, mixed number, or fraction greater than 1 be represented as the sum or difference of whole numbers and/or fractions?

M.P.2. Reason abstractly and quantitatively. Attend to the meaning of a quantity, and decompose fractions in a variety of ways. For example, $\frac{12}{5}$ can be decomposed in many different ways: $\frac{5}{5}+\frac{5}{5}+\frac{2}{5}$; $2+\frac{2}{5}$; $\frac{6}{5}+\frac{6}{5}$; $\frac{1}{5}+\frac{1}{5}+1+\frac{5}{5}$; $\frac{15}{5}-\frac{3}{5}$; etc.

Key Academic Terms:

fraction, sum, difference, equivalence, compose, decompose, mixed number, whole number, numerator, denominator, like denominators, equation

Number and Operations – Fractions
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.14 Understand a fraction $\frac{a}{b}$ with *a* > 1 as a sum of fractions $\frac{1}{b}$. **4.NF.14c** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

Guiding Question with Connections to Mathematical Practices:

How can mixed numbers with like denominators be added or subtracted in a variety of ways?

M.P.1. Make sense of problems and persevere in solving them. Apply a variety of strategies to solve addition and subtraction problems with like denominators. For example, $3\frac{1}{6} - \frac{4}{6} = (2 + \frac{6}{6} + \frac{1}{6}) - \frac{4}{6} = 2\frac{7}{6} - \frac{4}{6} = 2\frac{3}{6}$.

Key Academic Terms:

fraction, addition, subtraction, mixed number, equivalent fraction, whole number, numerator, denominator, like denominators, decompose

Number and Operations – Fractions
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.14 Understand a fraction $\frac{a}{b}$ with *a* > 1 as a sum of fractions $\frac{1}{b}$. **4.NF.14d** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Guiding Question with Connections to Mathematical Practices:

How can a fraction word problem be represented with an equation and visual model?

M.P.4. Model with mathematics. Explain connections between verbal descriptions, equations/expressions, and visual models of fraction word problems. For example, consider the following word problem: "Angel walked a total of $2\frac{1}{8}$ miles on Monday and Tuesday. Angel walked $\frac{5}{8}$ of a mile on Tuesday. How many miles did Angel walk on Monday?" This word problem connects to the expression $2\frac{1}{8} - \frac{5}{8}$. The expression is solved by using a fraction model, where one of the wholes is decomposed to use in subtraction. An equation is written to represent the fraction model: $1 + \frac{1}{8} + \frac{8}{8} - (\frac{4}{8} + \frac{1}{8}) = 1 + (\frac{1}{8} - \frac{1}{8}) + (\frac{8}{8} - \frac{4}{8}) = 1\frac{4}{8}$. The equation is then contextualized to state that Angel walked $1\frac{4}{8}$ miles on Monday.

Key Academic Terms:

fraction, word problem, visual model, mixed number, whole number, numerator, denominator, like denominators, expression, equation, decompose

Number and Operations – Fractions
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.15 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. **4.NF.15a** Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. Example: Use a visual fraction model to represent $\frac{5}{4}$ as the product 5 × ($\frac{1}{4}$), recording the conclusion by the equation $\frac{5}{4}$ = 5 × ($\frac{1}{4}$).

Guiding Questions with Connections to Mathematical Practices:

How is multiplying a fraction by a whole number similar to and different from multiplying two whole numbers?

M.P.7. Look for and make use of structure. Explain the similarities and differences between multiplication of whole numbers and multiplication of a fraction by a whole number. For example, the meaning of multiplication stays the same; multiplication is always represented by either a multiplicative comparison or "groups of" numbers which represent repeated addition.

How can a fraction be decomposed and composed into a product of a whole number times a unit fraction?

M.P.5. Use appropriate tools strategically. Use visual models in a variety of ways to show how to multiply a unit fraction by a whole number. For example, demonstrate multiplication of $\frac{1}{8} \times 3$ using methods such as skip-counting on a number line (e.g., "one eighth, two eighths, three eighths"), or drawing pictures of $\frac{1}{8}$ three times.

M.P.2. Reason abstractly and quantitatively. Attend to the meaning of a quantity and decompose fractions using visual models to write equations with repeated addition and show the equivalent multiplication equation. For example, $\frac{4}{3}$ can be decomposed and written as $\frac{4}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 4 \times \frac{1}{3}$.

How does the fraction $\frac{a}{b}$ compare with the fraction $\frac{1}{b}$?

M.P.8. Look for and express regularity in repeated reasoning. Notice the generalization that $\frac{a}{b}$ is a multiple of $\frac{1}{b}$ after numerous experiences decomposing various fractions and mixed numbers. For example, given the fraction $\frac{6}{8}$, immediately recognize it can be decomposed into $6 \times \frac{1}{8}$.

Key Academic Terms:

unit fraction, numerator, denominator, equation, product, multiple, multiply, whole number, mixed number, decompose

Number and Operations – Fractions
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.15 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. **4.NF.15b** Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. Example: Use a visual fraction model to express 3 × $\left(\frac{2}{5}\right)$ as 6 × $\left(\frac{1}{5}\right)$, recognizing this product as $\frac{6}{5}$. (In general, n × $\left(\frac{a}{b}\right)$ = $\frac{(n \times a)}{b}$.)

Guiding Question with Connections to Mathematical Practices:

How can fractions be multiplied by whole numbers in a variety of ways?

M.P.5. Use appropriate tools strategically. Use visual models, equations, and the properties of operations to decompose and compose numbers to solve the multiplication of a fraction and a whole number. For example, solve $\frac{7}{3} \times 2$ by decomposing $\frac{7}{3} = 7 \times \frac{1}{3}$ and then multiplying $2 \times 7 \times \frac{1}{3}$ to find $14 \times \frac{1}{3} = \frac{14}{3}$.

M.P.8. Look for and express regularity in repeated reasoning. Notice the generalization, after many examples of multiplying a whole number by a fraction, that $n \times \left(\frac{a}{b}\right) = \frac{(n \times a)}{b}$. For example, given the problem $8 \times \frac{2}{3}$, immediately recognize that it can be solved as $\frac{8 \times 2}{3} = \frac{16}{3}$.

Key Academic Terms:

unit fraction, numerator, denominator, equation, product, multiple, multiply, whole number, mixed number, compose, decompose

Number and Operations – Fractions
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.15 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. **4.NF.15c** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. Example: If each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between which two whole numbers does your answer lie?

Guiding Question with Connections to Mathematical Practices:

How can a fraction word problem be represented with an equation and visual model?

M.P.1. Make sense of problems and persevere in solving them. Explain connections between verbal descriptions, equations/expressions, and visual models of fraction word problems. For example, consider the following word problem: "A recipe calls for $1\frac{3}{4}$ cups of flour. Javier triples the recipe. How much flour does Javier use?" This word problem connects to the expression $1\frac{3}{4} \times 3$. The expression is represented by drawing $1\frac{3}{4}$ rectangles three times. An equation is written to represent the model: $(1 + 1 + 1) + (\frac{3}{4} + \frac{3}{4} + \frac{3}{4}) = (3 \times 1) + (3 \times \frac{3}{4}) = 3 + \frac{9}{4} = 3 + \frac{4}{4} + \frac{4}{4} + \frac{1}{4} = 5\frac{1}{4}$. The equation is then contextualized to state that Javier uses $5\frac{1}{4}$ cups of flour.

Key Academic Terms:

unit fraction, numerator, denominator, equation, product, multiple, multiply, whole number, mixed number

Number and Operations – Fractions
Understand decimal notation for fractions, and compare decimal fractions.
4.NF.16 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.) Example: Express $\frac{3}{10}$ as $\frac{30}{100}$, and add $\frac{3}{10}$ + $\frac{4}{100}$ = $\frac{34}{100}$.

Guiding Questions with Connections to Mathematical Practices:

How can equivalent fractions be created for fractions with 10 and 100 in the denominator?

M.P.5. Use appropriate tools strategically. Convert a fraction with denominator 10 to an equivalent fraction of denominator 100 by using fraction models such as base-ten blocks or hundredths grids. For example, represent $\frac{2}{10}$ using two long base-ten blocks and demonstrate that the quantity is equivalent to 20 small base-ten cubes.

How can two fractions with denominators 10 and 100 be added?

M.P.4. Model with mathematics. Apply more than one strategy for adding tenths and hundredths. For example, to add $\frac{4}{10} + \frac{11}{100}$, use base-ten blocks to show $\frac{4}{10} = \frac{40}{100}$ and add $\frac{40}{100} + \frac{11}{100}, = \frac{51}{100}$. Then connect the visual representation with an equation.

Key Academic Terms:

fractions, fraction models, tenths, hundredths, numerator, denominator, addition, hundredths grid, equivalent fraction

Number and Operations – Fractions
Understand decimal notation for fractions, and compare decimal fractions.
4.NF.17 Use decimal notation for fractions with denominators 10 or 100. Example: Rewrite 0.62 as $\frac{62}{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

Guiding Questions with Connections to Mathematical Practices:

How can fractions with denominators of 10 or 100 be written in decimal notation?

M.P.8. Look for and express regularity in repeated reasoning. Apply place value patterns for relating fractions with denominators of 10 and 100 to decimals. For example, notice that tenths written as fractions have "one" zero in the denominator and as decimals are located "one" space to the right of the decimal point (e.g., 2.8 has one digit after the decimal point, so it is written in fraction form as $2\frac{8}{10}$).

How does place value language extend to decimals to describe decimals in a variety of ways?

M.P.6. Attend to precision. Connect whole number place value language to decimals and use place value language to describe decimals in a variety of ways. For example, describe 0.13 as "one tenth and three hundredths," "thirteen hundredths," "zero point one three," and "point one three," but not "zero point thirteen." Note that using "point" doesn't use place value thinking, but it is not uncommon for mathematicians and scientists to use "point" when describing decimals.

How are decimals used in the real world?

M.P.2. Reason abstractly and quantitatively. Represent decimals using symbols on paper and understand what they mean. For example, an amount of money (35 cents), a distance (35 hundredths of a kilometer), and an amount of time (35 hundredths of a second faster than another runner) all can be represented on paper as 0.35.

How can decimals be located on a number line?

M.P.5. Use appropriate tools strategically. Locate decimals on a number line. For example, demonstrate how a meter stick can be used as a number line, representing a whole of 100 centimeters or 1 meter, to show that 16 centimeters is equivalent to 16 hundredths of a meter (as well as 1 tenth and 6 hundredths of a meter) and can be written as 0.16 or $\frac{16}{100}$.

Key Academic Terms:

fraction, numerator, denominator, decimal notation, tenths, hundredths, decimal point, meter, centimeter

Number and Operations – Fractions
Understand decimal notation for fractions, and compare decimal fractions.
4.NF.18 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Guiding Questions with Connections to Mathematical Practices:

What strategies can be used to determine the size of a decimal and compare it to another decimal?

M.P.5. Use appropriate tools strategically. Extend place value understand from whole numbers to demonstrate how to compare decimals that are in the tenths of hundredths place by using models such as base-ten blocks, hundredth grids, number lines, or equivalent number forms. For example, show that 0.2 is greater than 0.11 by shading each value on a hundredths grid.

When is it appropriate to compare two decimals?

M.P.2. Reason abstractly and quantitatively. Understand that when comparing decimals, the decimals must refer to the same whole. For example, if one hundredths grid is 10 centimeters by 10 centimeters and another hundredths grid is 10 inches by 10 inches, the wholes are not the same size, so it would not be appropriate to compare decimals related to the two different sized grids.

How can two decimals be compared using symbols?

M.P.4. Model with mathematics. Record the results of the comparison of two decimals by using the mathematical symbols $>$, $<$, or $=$. For example, $0.6 > 0.44$.

Key Academic Terms:

compare, greater than, less than, equals, tenths, hundredths, whole, hundredths grid

Measurement and Data
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
4.MD.19 Know relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz; l, ml; and hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. Examples: Know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...

Guiding Questions with Connections to Mathematical Practices:

How can the relative size of measurement units within a system of units be compared?

M.P.7. Look for and make use of structure. Identify personal benchmarks to learn and practice relative sizes. For example, a paperclip has a mass of approximately 1 gram, which can be used as the benchmark to estimate the mass of other objects.

M.P.8. Look for and express regularity in repeated reasoning. Notice patterns in metric prefixes that help relate one quantity to another, and connect the metric system to place value. For example, “kilo” means one thousand, so a kilometer is 1,000 meters and a kilogram is 1,000 grams.

How can tables be used to record relative measurements?

M.P.4. Model with mathematics. Record examples of equivalent measurements in a two-column table. For example, for a table with centimeters and meters, use the rule “divide by 100” when converting centimeters to meters and “multiply by 100” when converting meters to centimeters.

Key Academic Terms:

personal benchmark, relative size, prefix, equivalent measurement, conversion table, estimate, mass, metric system, comparisons, kilometer, meter, centimeter, gram, pound, ounce, liter, milliliter, hour, minute, second

Measurement and Data
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
4.MD.20 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Guiding Question with Connections to Mathematical Practices:

How can measurement word problems be represented and solved?

M.P.1. Make sense of problems and persevere in solving them. Write equations to solve measurement problems. For example, given the problem "A recipe for one loaf of bread uses $\frac{1}{4}$ cup of sugar. How many cups of sugar are used to make 3 loaves of bread?" write the equation $\frac{1}{4} \times 3 = c$, and $c = \frac{3}{4}$ cups of sugar.

M.P.4. Model with mathematics. Draw and use diagrams to solve measurement problems. For example, given the problem "Patrick walks $\frac{4}{10}$ of a kilometer. Angela walks 3 times farther than Patrick walks. How many more meters does Angela walk than Patrick walks?" use a double number line to show the connection between kilometers and meters and show how to use multiplication and subtraction to solve the problem.

Key Academic Terms:

operations, distance, intervals of time, liquid volume, mass, money, number line diagram, table, measurement scale

Measurement and Data
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
4.MD.21 Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. Example: Find the width of a rectangular room given the area of the flooring and the length by viewing the area formula as a multiplication equation with an unknown factor.

Guiding Questions with Connections to Mathematical Practices:

How can area and perimeter formulas be used to create and solve equations with an unknown?

M.P.7. Look for and make use of structure. Represent area and perimeter problems as equations with one unknown. For example, given the problem "A rectangle has a perimeter of 14 units and a length of 3 units. What is the width of the rectangle?" write and solve the equation that represents the problem $14 = 3 + 3 + w + w$ by recognizing that perimeter is the addition of the four sides of a rectangle.

How can visual models be used to apply the area and perimeter formulas?

M.P.4. Model with mathematics. Draw visual representations of rectangles to apply the area and perimeter formulas. For example, given the problem "A rectangle-shaped playground has an area of 56 square meters and a width of 7 meters. What is the length of the playground?" draw a sketch of the rectangle to determine that the length and width are multiplied when finding area, so $56 = L \times 7$; therefore, $L = 8$ meters.

Key Academic Terms:

area, perimeter, formula, rectangle, length, width, equation, square units

Measurement and Data
Represent and interpret data.
4.MD.22 Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots. Example: From a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.

Guiding Questions with Connections to Mathematical Practices:

How is a line plot used to display a set of data?

M.P.6. Attend to precision. Create a line plot to represent measurement data, paying attention to the key features of a line plot. For example, measure and record the amount of precipitation for one month and then represent the collected data by creating a line plot using the correct scale, title, and label.

How can the data in a line plot be analyzed to solve real-world problems?

M.P.2. Reason abstractly and quantitatively. Analyze the line plot in context to solve addition and subtraction problems. For example, find the difference between the minimum and maximum values of a data set representing the thumb lengths of the fourth graders in a classroom if the minimum value is the shortest thumb length and the maximum value is the longest thumb length.

Key Academic Terms:

line plot, measurement, fraction, mixed number, data set, visual model, maximum, minimum, scale, title, label

Measurement and Data
Geometric measurement: understand concepts of angle and measure angles.
4.MD.23 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. **4.MD.23a** An angle is measured with reference to a circle with its center at the common endpoint of the rays by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a “one-degree angle” and can be used to measure angles.

Guiding Questions with Connections to Mathematical Practices:

How are angles measured in reference to a circle?

M.P.4. Model with mathematics. Connect the definition of an angle to a circle. For example, the vertex of an angle is the same point as the center of a circle.

What is the fraction of a circle that makes a one-degree angle, and how is it used?

M.P.6. Attend to precision. Identify that a one-degree angle is equal to $\frac{1}{360}$ of a circle and that a one-degree angle being worth $\frac{1}{360}$ can be used to measure angles. For example, an angle that turns 1° takes up $\frac{1}{360}$ of the whole circle.

Key Academic Terms:

angle, reference, circle, center, endpoint, ray, intersect, fraction, arc, point, one-degree angle, vertex

Measurement and Data
Geometric measurement: understand concepts of angle and measure angles.
4.MD.23 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. **4.MD.23b** An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

Guiding Question with Connections to Mathematical Practices:

How are angles measured in terms of a specific number of one-degree angles?

M.P.2. Reason abstractly and quantitatively. Explain that the number of one-degree angles an angle turns through determines the measurement of the angle. For example, an angle that turns through 120 one-degree angles, or $\frac{120}{360}$ of the circle, measures 120°.

Key Academic Terms:

angle, one-degree angle, circle, center, endpoint

Measurement and Data
Geometric measurement: understand concepts of angle and measure angles.
4.MD.24 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

Guiding Questions with Connections to Mathematical Practices:

How are angles measured?

M.P.5. Use appropriate tools strategically. Demonstrate how to use a protractor to measure angles in different orientations to the nearest degree. For example, align the vertex of the angle with the correct point on the protractor, align one leg of the angle with the 0° mark on the protractor, and read where the other leg is located on the protractor.

How can angles be drawn to a given measurement?

M.P.5. Use appropriate tools strategically. Draw an angle of a given size using a variety of tools. For example, draw a ray on a piece of paper and use a protractor to construct a 108° angle.

Key Academic Terms:

measure, angle, degree, protractor, align, leg, ray, endpoint, point, vertex

Measurement and Data
Geometric measurement: understand concepts of angle and measure angles.
4.MD.25 Recognize angle measure as additive. When an angle is decomposed into nonoverlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world or mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Guiding Questions with Connections to Mathematical Practices:

How are angle measures additive?

M.P.7. Look for and make use of structure. Explain that angles can be decomposed into smaller angles and that the sum of the measurements of those angles equals the measurement of the larger angle. For example, a 180-degree angle is composed of 180 one-degree angles, or $180° = 60° + 30° + 90°$, or any other combination of angles that add up to 180°.

How can the additive nature of angle measures be used to solve real-world and mathematical problems?

M.P.2. Reason abstractly and quantitatively. Calculate the measurement of an angle composed of other angles. Find a missing angle measure when given the whole measure of an angle and the measure of part of the angle for real-world and mathematical problems. For example, for a 90° angle that has been decomposed into three angles, two of which measure 40° and 28°, the measurement of the third angle can be found by solving $90° - 40° - 28°$.

Key Academic Terms:

angle, additive, decompose, addition, subtraction, diagram, addend, sum, angle measure, equation, compose, nonoverlapping

Geometry
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
4.G.26 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

Guiding Questions with Connections to Mathematical Practices:

What are the characteristics of points, lines, line segments, rays, acute angles, right angles, obtuse angles, perpendicular lines, and parallel lines?

M.P.6. Attend to precision. Describe the characteristics of the given figures. For example, an obtuse angle is two rays that meet at a point called a vertex with an angle measure greater than 90 degrees.

How can points, lines, line segments, rays, acute angles, right angles, obtuse angles, perpendicular lines, and parallel lines be drawn?

M.P.5. Use appropriate tools strategically. Draw a given figure correctly using a variety of tools. For example, use a ruler, paper, and pencil to draw two points and connect them to make a line segment.

How can points, lines, line segments, rays, acute angles, right angles, obtuse angles, perpendicular lines, and parallel lines be distinguished in two-dimensional shapes?

M.P.6. Attend to precision. Identify the given figures in two-dimensional shapes. For example, in rectangle *ABCD*, identify that angle *ABC* is a right angle and that lines *AB* and *CD* are parallel.

Key Academic Terms:

point, line, line segment, ray, angle, right, acute, obtuse, perpendicular, parallel, two-dimensional, characteristics, vertex, naming conventions (i.e. rectangle ABCD or triangle ABC)

Geometry
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
4.G.27 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

Guiding Questions with Connections to Mathematical Practices:

How can angle sizes, parallel lines, and/or perpendicular lines be used to categorize two-dimensional shapes?

M.P.7. Look for and make use of structure. Sort two-dimensional figures based on angle sizes or presence of parallel and/or perpendicular lines. For example, given a group of regular polygons, sort shapes into categories based on angle size as well as presence of parallel lines.

M.P.6. Attend to precision. Classify and name shapes using more than one characteristic. For example, a four-sided shape with opposite sides parallel and four right angles is both a parallelogram and a rectangle. A quadrilateral with opposite sides parallel and four congruent sides is both a parallelogram and a rhombus.

What are the characteristics of a right triangle?

M.P.7. Look for and make use of structure. Identify right triangles. Describe a triangle with one right angle and two acute angles as a right triangle. For example, given a group of triangles, identify which triangles are right triangles by looking for the triangles that have one right angle.

Key Academic Terms:

classify, categorize, characteristics, two-dimensional, polygon, angle, acute, obtuse, right angle, scalene, isosceles, equilateral, equiangular, parallel, perpendicular, regular polygon, parallelogram, rhombus

Geometry
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
4.G.28 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Guiding Question with Connections to Mathematical Practices:

How can a line of symmetry be identified in a figure?

M.P.6. Attend to precision. Define, identify, and draw lines of symmetry by folding a shape. For example, fold a square in half four different ways to draw its four lines of symmetry. Also, for any shape, fold the paper in half to see if both halves are equal or are mirror images.

M.P.4. Model with mathematics. Given half of a two-dimensional figure on grid paper, draw the other half so that the two sides are symmetrical. Draw the line of symmetry that separates the two halves. For example, given half of a picture of a four-leaf clover, draw the other half of the picture.

Key Academic Terms:

symmetry, line of symmetry, two-dimensional figure, equal halves, mirror image